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# Elliptical galaxies interacting with the cluster tidal field: origin of the intracluster stellar population

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**Abstract.**
With the aid of simple numerical models, we discuss a particular aspect of the interaction between stellar orbital periods inside elliptical galaxies (Es) and the parent cluster tidal field (CTF), i.e., the possibility that *collisionless stellar evaporation* from Es is an effective mechanism for the production of the recently discovered intracluster stellar populations (ISP). These very preliminary investigations, based on idealized galaxy density profiles (such as Ferrers density distributions) show that, over an Hubble time, the amount of stars lost by a representative galaxy may sum up to the 10% of the initial galaxy mass, a fraction in interesting agreement with observational data. The effectiveness of this mechanism is due to the fact that the galaxy oscillation periods near its equilibrium configurations in the CTF are of the same order of stellar orbital times in the external galaxy regions.

**Key words.** Clusters: Galaxies – Galaxies: Ellipticals – Stellar Dynamics: Collisionless systems

## 1. Introduction

Observational evidences of an Intracluster Stellar Population (ISP) are mainly based on the identification of *intergalactic* planetary nebulae and red giant branch stars (see, e.g., Theuns & Warren 1996, Mèndez et al. 1998, Feldmeier et al. 1998, Arnaboldi et al. 2002, Durrell et al. 2002). Overall, the data suggest that approximately 10% (or even more) of the stellar mass of the cluster is contributed by the ISP (see, e.g., Ferguson & Tanvir 1998).

The usual scenario assumed to explain the finding above is that gravitational interactions between galaxies in cluster and/or interactions between the galaxies with the tidal gravitational field of the parent cluster (CTF), lead to a substantial stripping of stars from the galaxies themselves.

Here, supported by a curious coincidence, namely by the fact that *the characteristic times of oscillation of a galaxy around its equilibrium position in the CTF are of the same order of magnitude of the stellar orbital periods in the external part of the galaxy itself*, we suggest that an additional effect is at work, i.e. we discuss about the possible "resonant" interaction between stellar orbits inside the galaxies and the CTF.

In fact, based on the observational evidence that the major axis of cluster Es seems to be preferentially oriented toward the cluster center, N-body simulations showed that galaxies tend to align reacting to the CTF as rigid (Ciotti & Dutta 1994). By assuming this idealized scenario, a stability analysis then shows that this configuration is of equilibrium, and allows to calculate the oscillation periods in the linearized regime (Ciotti & Giampieri 1998, hereafter CG98).

Prompted by these observational and theoretical considerations, in our numerical explorations we assume that the galaxy is a triaxial ellipsoid with its center of mass at rest at the center of a triaxial cluster. The considerably more complicate case of a galaxy with the center of mass in rotation around the center of a spherical cluster will be discussed elsewhere (Ciotti & Muccione 2003, hereafter CM03).

## 2. The physical background

Without loss of generality we assume that in the (inertial) Cartesian coordinate system $C$ centered on the cluster center, the CTF tensor $\boldsymbol{T}$ is in diagonal form, with components $T_i$ ($i = 1, 2, 3$). By using three successive, counterclockwise rotations ($\varphi$ around $x$ axis, $\vartheta$ around $y'$ axis and $\psi$ around $z''$ axis), the linearized equations of motion for the galaxy near the equilibrium configurations can be written as

$$\ddot{\varphi} = \frac{\Delta T_{32}\Delta I_{32}}{I_1}\varphi,$$
$$\ddot{\vartheta} = \frac{\Delta T_{31}\Delta I_{31}}{I_2}\vartheta,$$
$$\ddot{\psi} = \frac{\Delta T_{21}\Delta I_{21}}{I_3}\psi, \qquad (1)$$

where $\Delta T$ is the *antisymmetric* tensor of components $\Delta T_{ij} \equiv T_i - T_j$, and $I_i$ are the principal components of the galaxy inertia tensor. In addition, let us also assume that $T_1 \geq T_2 \geq T_3$ and $I_1 \leq I_2 \leq I_3$, i.e., that $\Delta T_{32}, \Delta T_{31}$ and $\Delta T_{21}$ are all less or equal to zero (see Section 3). Thus, the equilibrium position of 1 is *linearly stable*, and its solution is

$$\varphi = \varphi_{\rm M}\cos(\omega_\varphi t),$$
$$\vartheta = \vartheta_{\rm M}\cos(\omega_\vartheta t),$$
$$\psi = \psi_{\rm M}\cos(\omega_\psi t), \qquad (2)$$

where

$$\omega_\varphi = \sqrt{\frac{\Delta T_{23}\Delta I_{32}}{I_1}},$$
$$\omega_\vartheta = \sqrt{\frac{\Delta T_{13}\Delta I_{31}}{I_2}},$$
$$\omega_\psi = \sqrt{\frac{\Delta T_{12}\Delta I_{21}}{I_3}}. \qquad (3)$$

For computational reasons the best reference system in which calculate stellar orbits is the (non inertial) reference system $C'$ in which the galaxy is at rest, and its inertia tensor is in diagonal form. The equation of the motion for a star in $C'$ is

$$\ddot{\boldsymbol{x}}' = \mathcal{R}^{\rm T}\ddot{\boldsymbol{x}} - 2\boldsymbol{\Omega}\wedge\boldsymbol{v}' - \dot{\boldsymbol{\Omega}}\wedge\boldsymbol{x}' - \boldsymbol{\Omega}\wedge(\boldsymbol{\Omega}\wedge\boldsymbol{x}'), \qquad (4)$$

where $\boldsymbol{x} = \mathcal{R}(\varphi, \vartheta, \psi)\boldsymbol{x}'$, and

$$\boldsymbol{\Omega} = (\dot{\varphi}\cos\vartheta\cos\psi + \dot{\vartheta}\sin\psi, -\dot{\varphi}\cos\vartheta\sin\psi + \dot{\vartheta}\cos\psi, \dot{\varphi}\sin\vartheta + \dot{\psi}). \qquad (5)$$

In eq. (4)

$$\mathcal{R}^{\rm T}\ddot{\boldsymbol{x}} = -\nabla_{\boldsymbol{x}'}\phi_{\rm g} + (\mathcal{R}^{\rm T}\boldsymbol{T}\mathcal{R})\boldsymbol{x}', \qquad (6)$$

where $\phi_{\rm g}(\boldsymbol{x}')$ is the galactic gravitational potential, $\nabla_{\boldsymbol{x}'}$ is the gradient operator in $C'$, and we used the tidal approximation to obtain the star acceleration due to the cluster gravitational field.

## 3. Period estimations

For simplicity, in the following estimates we assume that the galaxy and cluster densities are stratified on homeoids. In particular, we use a galaxy density profile belongings to the ellipsoidal generalization of the widely used $\gamma$-models (Dehnen 1993, Tremaine et al. 1994):

$$\rho_{\rm g}(m) = \frac{M_{\rm g}}{\alpha_1\alpha_2\alpha_3}\frac{3-\gamma}{4\pi}\frac{1}{m^{\gamma}(1+m)^{4-\gamma}}, \quad (7)$$

where $M_{\rm g}$ is the total mass of the galaxy, $0 \leq \gamma \leq 3$, and

$$m^2 = \sum_{i=1}^{3}\frac{(x_i')^2}{\alpha_i^2}, \qquad \alpha_1 \geq \alpha_2 \geq \alpha_3. \quad (8)$$

The inertia tensor components for this family are given by

$$I_i = \frac{4\pi}{3}\alpha_1\alpha_2\alpha_3(\alpha_j^2+\alpha_k^2)h_{\rm g}, \quad (9)$$

where $h_{\rm g} = \int_0^\infty \rho_{\rm g}(m)m^4 dm$, and so $I_1 \leq I_2 \leq I_3$. Note that, from eq. (3), *the frequencies for homeoidal stratifications do not depend on the specific density distribution assumed, but only on the quantities* $(\alpha_1, \alpha_2, \alpha_3)$. We also introduce the two ellipticities

$$\frac{\alpha_2}{\alpha_1} \equiv 1-\epsilon, \qquad \frac{\alpha_3}{\alpha_1} \equiv 1-\eta, \quad (10)$$

where $\epsilon \leq \eta \leq 0.7$ in order to reproduce realistic flattenings.

A rough estimate of *characteristic stellar orbital times* inside $m$ is given by $P_{orb}(m) \simeq 4P_{{\rm d}yn}(m) = \sqrt{3\pi/G\overline{\rho_{\rm g}}(m)}$, where $\overline{\rho_{\rm g}}(m)$ is the mean galaxy density inside $m$. We thus obtain

$$P_{orb}(m) \simeq 9.35\times 10^6\sqrt{\frac{\alpha_{1,1}^3(1-\epsilon)(1-\eta)}{M_{\rm g,11}}} \times m^{\gamma/2}(1+m)^{(3-\gamma)/2}\ {\rm yrs}, \quad (11)$$

where $M_{\rm g,11}$ is the galaxy mass normalized to $10^{11}M_{\odot}$, $\alpha_{1,1}$ is the galaxy "core" major axis in kpc units (for the spherically symmetric $\gamma = 1$ Hernquist 1990 models, $R_{\rm e} \simeq 1.8\alpha_1$); thus, in the outskirts of normal galaxies orbital times well exceeds $10^8$ or even $10^9$ yrs.

For the cluster density profile we assume

$$\rho_{\rm c}(m) = \frac{\rho_{\rm c,0}}{(1+m^2)^2}, \quad (12)$$

where $m$ is given by an identity similar to eq. (8), with $a_1 \geq a_2 \geq a_3$, and, in analogy with eq. (10), we define $a_2/a_1 \equiv 1-\mu$ and $a_3/a_1 \equiv 1-\nu$, with $\mu \leq \nu \leq 1$.

It can be shown that the CTF components at the center of a non-singular homeoidal distribution are given by

$$T_i = -2\pi G\rho_{\rm c,0}w_i(\mu,\nu), \quad (13)$$

where the dimensionless quantities $w_i$ are independent of the specific density profile, $w_1 \leq w_2 \leq w_3$ for $a_1 \geq a_2 \geq a_3$, and so the conditions for stable equilibrium in eq. (1) are fulfilled (CG98, CM03).

The quantity $\rho_{\rm c,0}$ is not a well measured quantity in real clusters, and for its determination we use the virial theorem, $M_{\rm c}\sigma_{\rm V}^2 = -U$, where $\sigma_{\rm V}^2$ is the virial velocity dispersion, that we assume to be estimated by the observed velocity dispersion of galaxies in the cluster. Thus, we can now compare the galactic oscillation periods (for small galaxy and cluster flattenings, see CM03)

$$\begin{aligned} P_{\varphi} &= \frac{2\pi}{\omega_{\varphi}} \simeq \frac{8.58\times10^8}{\sqrt{(\nu-\mu)(\eta-\varepsilon)}}\frac{a_{1,250}}{\sigma_{\rm V,1000}}\ {\rm yrs}\ , \\ P_{\vartheta} &= \frac{2\pi}{\omega_{\vartheta}} \simeq \frac{8.58\times10^8}{\sqrt{\nu\eta}}\frac{a_{1,250}}{\sigma_{\rm V,1000}}\ {\rm yrs}\ , \\ P_{\psi} &= \frac{2\pi}{\omega_{\psi}} \simeq \frac{8.58\times10^8}{\sqrt{\mu\varepsilon}}\frac{a_{1,250}}{\sigma_{\rm V,1000}}\ {\rm yrs}\ . \end{aligned} \quad (14)$$

(where $a_{1,250} = a_1/250{\rm kpc}$, and $\sigma_{\rm V,1000} = \sigma_{\rm V}/10^3{\rm km\ s^{-1}}$) with the characteristic orbital times in galaxies. From eqs. (11) and (14), it follows that *in the outer halo of giant Es the stellar orbital times can be of the same order of magnitude as the oscillatory periods of the galaxies themselves in the CTF.* For example, in a relatively small galaxy of $M_{\rm g,11} = 0.1$ and $\alpha_{1,1} = 1$,

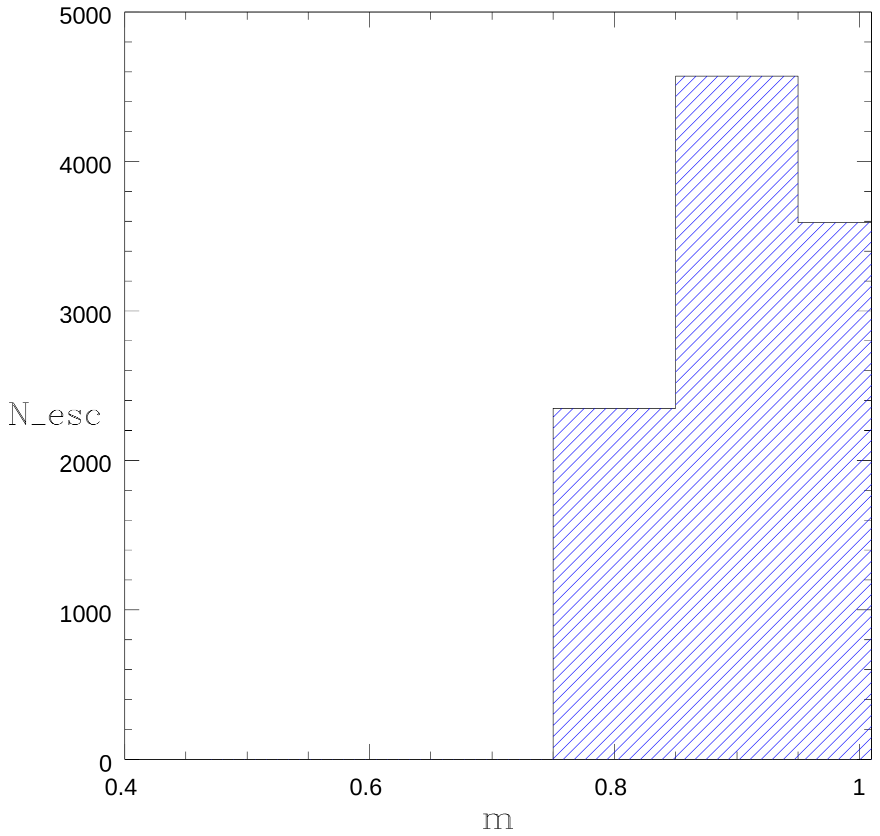


**Fig. 1.** Histogram of $m$ at $t = 0$ vs. $N_{\rm esc}$ for a galaxy of $M_{\rm g} = 10^{10}\,M_{\odot}$ and $\alpha_1 = 20$ kpc, with $\epsilon = 0.2$ and $\eta = 0.4$. The cluster parameters are $\mu = 0.2$, $\nu = 0.4$, $a_{1,250} = \sigma_{\rm V,1000} = 1$, and the galaxy maximum oscillations angles in eqs. (2) are fixed to 0.2 rad. In this simulation $N_{\rm tot} = 10^5$.

$P_{orb} \simeq 1$ Gyr is at $m \simeq 10$ (i.e., at $\simeq 5R_{\rm e}$), while for a galaxy with $M_{\rm g,11} = 1$ and $\alpha_{1,1} = 3$ the same orbital time is at $m \simeq 7$ (i.e., at $\simeq 3.5R_{\rm e}$).

## 4. The numerical integration scheme

Here we present the preliminary results obtained by using only a very idealized galactic density profile, while more realistic galaxy density distributions, such as triaxial Hernquist models, are described elsewhere (CM03, Muccione & Ciotti 2003, hereafter MC03). In particular, here we explore the evaporation process in the case of a Ferrers (see, e.g., Binney & Tremaine

1987) models, with density profiles given by

$$\rho_g = \begin{cases} \rho_g(0)(1-m^2)^n & \text{for } m \leq 1 \\ 0 & \text{for } m > 1 \end{cases} \quad (15)$$

where $m$ is the homeoidal radius defined as in eq. (8). The case $n = 0$ corresponds to an anisotropic harmonic oscillator, while for $n \geq 0$ the density distribution is radially decreasing. A nice property of Ferrers models (for integer $n$) is that their gravitational potential can be simply expressed in algebraic form (see, e.g., Chandrasekhar 1969).

To integrate the second order differential equations (11) for each star, we use a code based on an adapted Runge-Kutta routine. The initial conditions are obtained by using the Von Neumann *rejection method*, and with this method we arrange $N_{\rm tot}$ initial conditions in configuration space which reproduce the density profile in eq. (15). In MC03 and CM03 the three components of the initial velocity for each star are assigned by considering the local velocity dispersion of the galaxy model at rest, while here, for simplicity, each star at $t = 0$ is characterized by null velocity. Within $t = t_{\rm H}$ (where $t_{\rm H} = 1.5 \times 10^{10}$ yrs), the code checks which of the initial conditions result in a orbit with $m > 1$. This escape condition is very crude, and a more sophisticated criterium is adopted in MC03 and CM03, where *untruncated* galaxy models are used. In standard simulations we use, as a rule, $N_{\rm tot} = 10^5$, thus, from this point of view, we are exploring $10^5$ independent "1-body problems" in a time–dependent force field.

The simulations are performed on the *GRAVITOR*, the Geneva Observatory 132 processors Beowulf cluster (**http://obswww.unige.ch/~pfennige/gravitor/gravitor_e.html**).

## 5. Preliminary results and conclusions

In Fig. 1 we show the results of one of our preliminary simulations for a galaxy model with $n = 1$, $M_{\rm g} = 10^{10} M_{\odot}$, semi-mayor axis $\alpha_1 = 20$ kpc, flattenings $\epsilon = 0.2$, $\eta = 0.4$, and maximum oscillation angles equals to 0.2 rad. The cluster parameters are $a_{1,250} = \sigma_{\rm V,1000} = 1$, $\mu = 0.2$, $\nu = 0.4$, and the total number of explored orbits is $N_{\rm tot} = 10^5$. In the ordinate axis we plot the number of *escapers* as a function of their homeoidal radius at $t = 0$. Note how the zone of maximum escape is near $m \simeq 0.8$, and how the total number of escapers is a significant fraction of the total number of explored orbits (actually, with the galaxy and cluster parameters adopted in the simulations, $N_{\rm esc} \simeq 0.1 N_{\rm tot}$). This number is a somewhat upper limit of the expected number in more realistic simulations: in fact, 1) as described in Section 4, we adopted a very weak escape criterium, and 2), in more realistic galaxy models the density decrease in the outer parts is stronger than in Ferrers models, and so we expect that a smaller number of stars will be significantly affected by the CTF. Both points are addressed and discussed elsewhere, in MC03 in a preliminary and qualitative way for an Hernquist model at the center of a cluster, while in CM03 we present the results of a systematic exploration of the parameter space for untruncated galaxies both at the center and with the center of mass in circular orbit in a spherical cluster.

In any case, the simple model here explored looks promising, with a number of escapers in nice agreement with the observational estimates.

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