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# A new determination of the Period–Radius relation for Classical Galactic Cepheids.

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**Abstract.** Using a modified version of the CORS Baade–Wesselink method in the Walraven system, we derive the radii of a sample of Galactic Cepheids with period from few days up to 40 days. We test our results by performing a comparison with the radii obtained from interferometric measurements, finding an excellent agreement. The best fit Period–Radius relation $\log R = (0.75 \pm 0.02) \log P + (1.10 \pm 0.03)$ is obtained by fixing the projection factor to p=1.27 and is in good agreement with those from the literature.

**Key words.** Cepheids – Baade–Wesselink – Period–Radius relation – Walraven

## 1. Introduction

Cepheids have a key role in the problem of determination of astronomical distances from Galactic scale up to 25 Mpc and provide an absolute calibration of secondary indicators used to estimate the value of the Hubble constant (Freedman et al. 2001).

The accuracy reached in the determination of cosmological distances with Cepheid–based methods is strictly connected with the knowledge of their structural parameters (luminosity, mass, radius, chemical composition) and of their pulsational mechanism (see e.g. Bono, Marconi & Stellingwerf 1999, 2000; Bono, Castellani & Marconi 2002; Keller & Wood 2006, and references therein).

In this work we focus our attention on the determination of the radii of Cepheids and on the calibration of the Period–Radius relation. Tipically the methods used to derive Cepheid radii from the photometric and spectroscopic data (radial velocity) are based on the classical Baade–Wesselink technique (Wesselink 1946). Here we use the CORS method, which consists in a powerful modfication of the Baade–Wesselink technique (Caccin et al. 1981). Using a new photometrically calibrated release by Pedicelli et al. (2009) of the time series originally published by Pel (1978) in the Walraven system (V, B, L, U, W), and the radial veloc-

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ity data available in the literature, we apply the CORS technique to a sample of $\sim 60$ Galactic Cepheids with period ranging from few days up to 40 days. The new photometric data, coupled with the updated model atmospheres by Castelli (1999), allow to derive precise stellar parameters (gravity and effective temperature) and to calibrate the surface brightness function, which is fundamental in the application of the CORS method (Onnembo et al. 1985).

## 2. The new CORS method

The basic equation of the classical CORS method can be obtained from a simple mathematical manipulation of the surface brightness function $S_V(\phi)$ (for details see e.g. Caccin et al. 1981):

$$q\int_0^1 \ln\left\{R_0 - pP\int_{\phi_0}^{\phi} v(\phi')d\phi'\right\}C'_{ij}d\phi + \\ -B + \Delta B = 0 \tag{1}$$

where $q = \frac{5}{\ln 10}$, $P$ is the period, $v$ is the radial velocity, $C_{ij}$ is a generic color, $\phi$ is the phase of the pulsational cycle, $p$ is the radial velocity projection factor and the other two terms are $B = \int_0^1 C_{ij}(\phi)m'_V(\phi)d\phi$ and $\Delta B = \int_0^1 C_{ij}(\phi)S'_V(\phi)d\phi$.

The projection factor $p$ relates the radial to the pulsational velocity and, to date, its uncertain value is the most important source of systematic errors in the radii derivation (see e.g Merand et al. 2005, for a review). In this work we test two possible values, the generally adopted value $p = 1.36$ (Ripepi et al. 1997) and a more recent derivation, $p = 1.27$ (Groenewegen 2007), obtained by using Cepheids HST parallaxes and interferometrically measured angular diameters.

The unknown quantity in Eq. (2) is the radius $R_0$ at an arbitrary phase $\phi_0$. It allows to derive the radius at any phase by integrating the radial velocity curve between $\phi_0$ and the $\phi$.

The term $B$, in Eq.(2), can be calculated by using color and magnitude, while the term $\Delta B$ is not directly connected to observational data because it contains the surface brightness, but as it has a small value, in the original CORS method it is set to zero (Caccin et al. 1981). However, the radii obtained by including this term are more accurate than those obtained from the original CORS method or using the classical Baade–Wesselink technique (Caccin et al. 1981; Ripepi et al. 1997; Ruoppo et al. 2004).

In order to calculate the $\Delta B$ term we have followed the same procedure adopted by Ruoppo et al. (2004). Under the approximation of quasi–static atmosphere for Cepheids (Onnembo et al. 1985), any photometric quantity can be expressed as a function of effec-

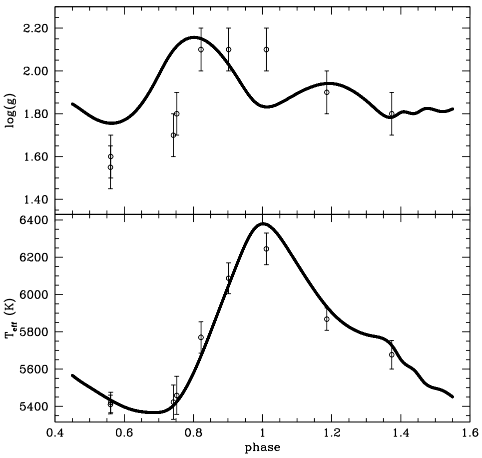


**Fig. 1.** Effective temperature curve (bottom panel) and gravity curve (upper panel), obtained by fitting the grid of model atmosphere, are plotted for the star U Sgr ($P = 7.9$ days). The open blue circles with the associated error bars represent the values of the physical parameters obtained by Luck & Andrievsky (2004) using spectroscopic data.

**Table 1.** Interferometric radii derived by Kervella et al. (2004) compared with the results of this work.

| Star | Kervella et al. | This work |
|---|---|---|
| $X\,Sgr$ | $52.2^{+23}_{-12}$ | $51.4^{+3.9}_{-3.6}$ |
| $\eta\,Aql$ | $59.3^{+5.3}_{-4.6}$ | $59.0^{+4.5}_{-4.2}$ |
| $W\,Sgr$ | $56.4^{+30}_{-16}$ | $53.4^{+4.1}_{-3.8}$ |
| $Y\,Oph$ | $136^{+325}_{-56}$ | $90.7^{+6.9}_{-6.4}$ |
| $l\,Car$ | $191.2^{+7.6}_{-6.0}$ | $194.6^{+14.9}_{13.8}$ |

**Table 2.** Coefficients of the Period–Radius relation from the literature and of our work. The first two columns contain the slope and the intercept of the linear relation, the third column contains the reference and the last one the technique used to derive the Period–Radius relation.

| $a$ | $b$ | $Source$ | $Method$ |
|---|---|---|---|
| 0.71 | 1.14 | Alibert et al. (1999) | Theory (Solar metallicity) |
| 0.75 ± 0.02 | 1.07 ± 0.02 | Gieren et al. (1998) | Surf. Brightness (variable p) |
| 0.74 ± 0.03 | 1.12 ± 0.03 | Rojo Arellano & Arellano Ferro (1994) | Surf. Brightness ($p$ = 1.31) |
| 0.606 ± 0.037 | 1.263 ± 0.033 | Ripepi et al. (1997) | Surf. Brightness ($\Delta B \neq 0$, $p$ = 1.36) |
| 0.767 ± 0.09 | 1.091 ± 0.011 | Kervella et al. (2004) | Interferometry ($p$ = 1.36) |
| 0.686 ± 0.036 | 1.134 ± 0.034 | Groenewegen (2007) | Interferometry ($p$ = 1.27) |
| 0.747 ± 0.028 | 1.071 ± 0.025 | Turner & Burke (2002) | Modified Baade–Wesselink ($p$ = 1.31) |
| 0.69 ± 0.09 | 1.18 ± 0.08 | Ruoppo et al. (2004) | New CORS ($\Delta B = 0$, $p$ = 1.36) |
| 0.74 ± 0.11 | 1.19 ± 0.09 | Ruoppo et al. (2004) | New CORS ($\Delta B \neq 0$, $p$ = 1.36) |
| 0.71 ± 0.03 | 1.16 ± 0.03 | This Work | New CORS ($\Delta B = 0$, $p$ = 1.36 ) |
| 0.75 ± 0.02 | 1.13 ± 0.03 | This Work | New CORS ($\Delta B \neq 0$, $p$ = 1.36) |
| 0.71 ± 0.03 | 1.13 ± 0.03 | This Work | New CORS ($\Delta B = 0$, $p$ = 1.27 ) |
| 0.75 ± 0.02 | 1.10 ± 0.03 | This Work | New CORS ($\Delta B \neq 0$, $p$ = 1.27) |

tive temperature and surface gravity: e.g. $S = S(T_e, g)$, $C_{ij} = C_{ij}(T_e, g)$, $C_{hk} = C_{hk}(T_e, g)$. From the availability of model atmosphere colors and from the inversion of the last two relations, we can express the surface brightness as a function of two colors: $S_V = S_V(C_{ij}, C_{hk})$. This parametrization of the surface brightness allows us to calculate the term $\Delta B$ in the CORS equation.

## 3. Test with spectroscopic and interferometric data

Using the observed colors (V-B) and (U-W) and the inverse relations $\log T_e = \log T_e(V-B, U-W)$ and $\log g = \log g(V-B, U-W)$, derived by fitting the grid of model atmospheres, we have estimated the structural parameters of Cepheids in our sample. In order to test the accuracy of this procedure, we have compared the derived effective temperature and the surface gravity for some Cepheids of our sample in common with other works, where structural parameters are derived from spectroscopic data.

As an example in Fig. 1 the effective temperature and gravity curves obtained in this work for the Cepheid U Sgr are plotted together with the values of the physical parameters obtained spectroscopically by Luck & Andrievsky (2004). The plot shows an excellent agreement between the curve of effective temperature and the spectroscopic data, while the agreement is less satisfactory for the curve of surface gravity, probably due to the deteriorated reflectivity in the W band at the epoch of observations (Lub & Pel 1977; Pel 1978), but this effect does not produce an uncertainty larger than 5% in the radius.

We have also compared the derived radii of some Cepheids of our sample with those obtained from interferometric measures by Kervella et al. (2004). Table 1 shows that the agreement between our values and interferometry is excellent.

## 4. The Period–Radius relation

In order to investigate the Period–Radius relation we have retained only those Cepheids classified as singular or visual binaries by using the results from Szabados (2003). Furtheremore, we have also excluded the first overtone pulsators using the classification by Fernie (1995). Using the radii of the final sample, containing 27 Cepheids, we have performed a weighted linear fit obtaining the results contained in Tab. 2, together with those from other authors.

In the table are reported the results obtained by including and excluding the $\Delta B$ term and by assuming $p = 1.27$ and $p = 1.36$.

Concerning the comparison with the theory (first line in Tab. 2) our relation without $\Delta B$ term, assuming $p = 1.27$, seems to provide a better agreement than the relation obtained including the $\Delta B$ term, which has a steeper slope and a smaller zero-point. If we consider the relation obtained with $p = 1.36$, we can observe that both the zero-points, with and without the $\Delta B$ term, are in agreement (within the uncertainties) with the theoretical one. As for the slope obtained excluding the $\Delta B$ term, it is in excellent agreement with the prediction of theory, while in the equation including the $\Delta B$ term the agreement is achieved only within $2\sigma$.

The main result derived from this work is that the agreement with the results of other authors increases by including the $\Delta B$ term and by decreasing the projection factor from 1.36 to 1.27.

In a subsequent work it will be interesting to investigate the implications of our finding on the Period–Luminosity relation and on the derivation of extragalactic distances (Molinaro et al. 2010, in preparation).

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