



Lithium diffusion after recombination

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Abstract. The observed amount of lithium along the Spite plateau disagrees by a factor of $\sim 3-5$ with the predictions of the standard cosmology. Since the observations are limited to the local Universe (halo stars, globular clusters and satellites of the Milky Way) it is possible that some physical processes led to the spatial separation of lithium and local reduction of $[\text{Li}/\text{H}]$. We study the question of lithium diffusion after the recombination in sub-Jeans halos, taking into account that more than 95% of lithium remains in the singly-ionized state at all times. Large scattering cross sections on the rest of the ionized gas leads to strong coupling of lithium to protons and its initial direction of diffusion coincides with that of H^+ . In the rest frame of the neutral gas this leads to the diffusion of H^+ and Li^+ out of overdensities depleting Li/H in the minima of gravitational wells relative to the primordial value, which may be a missing piece in the cosmological lithium puzzle.

Key words. recombination, atomic diffusion, Rutherford scattering

1. Introduction

The primordial abundances of light elements, ^4He , D , and ^7Li , offer unique window into the very early Universe at redshifts of $z \sim 10^9$. In recent years this probe has been sharpened: the only free parameter that enters the standard big bang nucleosynthesis (SBBN) calculations – baryon-to-photon ratio η_b – has been measured to great accuracy via the CMB experiments. With the improvements of the nuclear cross section measurements, the prediction for the primordial fraction of ^7Li is given by (Olive et al. 2012)

$$\left[\frac{^7\text{Li}}{\text{H}} \right]_{\text{SBBN}} = (5.07^{+0.71}_{-0.62}) \times 10^{-10}, \quad (1)$$

consistent with other determinations, $(5.24 \pm 0.5) \times 10^{-10}$ (Coc, A. et al. 2011). This pre-

diction is in strong contradiction with the Spite plateau value of ^7Li , $(1.23^{+0.34}_{-0.16}) \times 10^{-10}$ (Ryan et al. 2000), $(1.58 \pm 0.31) \times 10^{-10}$ (Sbordone et al. 2010), an observationally determined value of the lithium abundance in the atmospheres of hot PopII halo stars. For a long time this value was believed to be an objective measure of the primordial fraction of lithium in the Universe, and the fact that it is (3-5) times smaller than (1) is referred to as the lithium problem. It is far from clear whether regular astrophysical processes of lithium depletion could account for such a large deficit, and speculations of non-standard physics being behind the discrepancy flourished (see, *e.g.* reviews Cuoco et al. 2004; Jedamzik & Pospelov 2009; Pospelov & Pradler 2010; Fields 2011). Most recently, this problem have being further complicated by the

observation of the deterioration of the plateau at the lowest metallicities, $Z < 1.5 \times 10^{-3}$ (Sbordone et al. 2010; W. Aoki et al. 2009; Melendez et al. 2010). Since stellar physics cannot depend on Z in the limit of $Z \rightarrow 0$, these latest findings possibly point towards additional "missing" pieces of physics unrelated to the stellar physics, such as evolution of primordial gas leading to the formation of PopII stars with lowest metallicities.

It is important to realize that the observations of lithium abundance are done in the "local" Universe, while the SBBN predictions are global. It has been suggested that $O(1)$ downward fluctuation of η_b in the patch of the Universe that includes Milky Way may be responsible results in the reduced lithium abundance locally, $[^7\text{Li}/\text{H}]_{\text{local}} < [^7\text{Li}/\text{H}]_{\text{SBBN}}$ (Holder et al. 2010; Regis, & Clarkson 2012). While this idea clearly falls within "non-standard" cosmology scenarios, it calls for the re-evaluation of the *standard* physics processes that may lead to the local under- or over-abundance of lithium relative to the SBBN prediction. In the standard cosmological picture, the space fluctuations of $^7\text{Li}/\text{H}$ are initially small, $O(10^{-5})$, but consequently amplified by the growth of structure in combination with *diffusional processes* in the Early Universe.

The existing astrophysical literature covers the diffusion of elements in stellar atmospheres and in the cluster of galaxies (see, e.g. Aller & Chapman 1960; Michaud, G. et al. 1976; Thoul et al. 1994; Chuzhoy & Nusser 2003; L. Chuzhoy & Loeb 2004). In contrast, the studies of primordial element diffusion in the early Universe are very sparse. In the most complete study to date, Medvigy & Loeb (2001) have considered the evolution of elemental abundance in the linear regime, $\delta\rho/\rho \ll 1$. Although the linear regime by definition does not allow for large effects in the abundances, Medvigy & Loeb (2001) find that qualitative trend is such that lithium, owing to its larger mass, tends to accumulate more in the minima of gravitational potentials compared to hydrogen. Since the star formation should also occur inside gravitational wells, the qualitative trend inferred from Medvigy & Loeb (2001) is $[^7\text{Li}/\text{H}]_{\text{local}} > [^7\text{Li}/\text{H}]_{\text{SBBN}}$, which

does not help to solve the lithium problem in any way. While the treatment of most elements is correct, there is an important assumption made about the neutrality of lithium in Medvigy & Loeb (2001), which does not hold in the early Universe. In fact after the recombination, lithium exists predominantly in the singly-ionized state, $^7\text{Li}^+$ (Switzer & Hirata 2005). There are two reasons for that: firstly, the 5.39 eV ionization potential for lithium means that the recombination temperature is smaller than that of hydrogen by the factor of ~ 2.5 , at which time the density of the free electrons is depleted by two orders of magnitude, and the recombination rates are less than the Hubble expansion rate. Secondly, the nonequilibrium population of photons from residual $e - p$ recombination causes photo-ionization of neutral fraction and keeps its abundance below a few percent level throughout the cosmic history all the way to reionization at $z \sim 10$ (Switzer & Hirata 2005).

In this presentation we show that the fact that lithium remains in the $^7\text{Li}^+$ state has profound consequences for its diffusion after hydrogen recombination. In particular, we show that owing to the large scattering cross section on protons, $^7\text{Li}^+$ stays spatially bound to H^+ , and the direction of their diffusion is *against* the gravitational force in the reference frame where the neutral hydrogen is at rest. This is opposite to the direction of lithium diffusion inferred with the assumption of its neutrality. From our analysis it follows that $[^7\text{Li}/\text{H}]_{\text{grav min}} < [^7\text{Li}/\text{H}]_{\text{SBBN}}$, which can have deep implications for the cosmological lithium problem. In the rest of the paper, we expand on this observation in some detail, and further speculate on additional mechanisms that can lead to separation of charged and neutral particles in the cosmological setting.

2. Direction of lithium diffusion

We consider the equations for cosmological fluids of different primordial species with number densities n_a , where a spans H , ^4He , e , p , ^7Li . Note that after most of the hydrogen becomes neutral, the recombination rate for residual e and p is much smaller than the

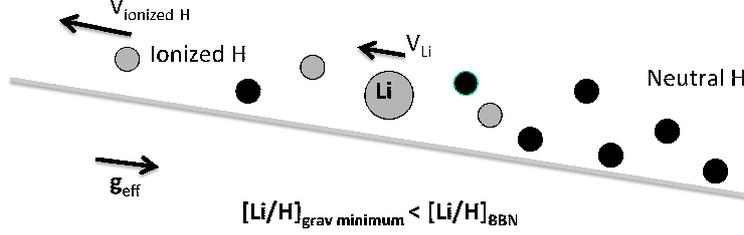


Fig. 1. Pictorial description to the main finding of the paper. In the rest frame of the neutral hydrogen (small black circles), combination of the gravitational and electromagnetic force leads to the diffusion of H^+ against the direction of gravitational acceleration \mathbf{g}_{eff} . Rutherford scattering of Li^+ on H^+ "overpowers" gravity of lithium and scattering on neutral hydrogen, and leads to the direction of Li^+ velocity to be directed against \mathbf{g}_{eff} . Therefore, in the minimum of the potential, $[\text{Li}/\text{H}]_{\text{grav min}} < [\text{Li}/\text{H}]_{\text{SBBN}}$.

Hubble expansion rate and they can be treated as separate species. The presence of small quantities of D and ^3He will not affect the evolution of ^7Li and can be neglected. With that in mind, we have a separate equations for the average velocities V_a of individual species,

$$\frac{\partial \mathbf{V}_a}{\partial t} \simeq \mathbf{g} - \frac{\nabla P_a}{\rho_a} + \frac{q_a}{m_a} \mathbf{E} - \sum_b \frac{\mathbf{V}_a - \mathbf{V}_b}{\tau_{ab}} + \frac{\mathbf{f}_a^{\text{ext}}}{m_a}. \quad (2)$$

In these equations, \mathbf{g} is the gravitational acceleration, \mathbf{E} is the electric field strength, P_a , $\rho_a = m_a n_a$ and q_a are the partial pressures and mass densities, and the electric charge. For the electromagnetic effects we assume the tight charge coupling approximation. The $\mathbf{V}_a - \mathbf{V}_b$ diffusion term is governed by the diffusion coefficients τ_{ab}^{-1} ,

$$\frac{1}{\tau_{ab}} = \frac{\mu_{ab}^2 n_b \langle \sigma_{ab} v^3 \rangle}{3T m_b}, \quad (3)$$

that are in turn determined by the transport cross sections σ_{ab} averaged over the microscopic velocity distribution. We use lower- and upper-case letters to distinguish between thermal v and diffusional \mathbf{V} velocities. μ_{ab} is reduced mass. T is the effective temperature of

the matter species that undergo thermal decoupling and become cooler than photon temperature T_γ at redshifts $z \sim 100$. Finally, the last term in (2) accounts for the possibility of additional external forces, such as radiation pressure, Lorentz force etc, with dependence on species index a . We take $\mathbf{f}_a^{\text{ext}} = 0$ for now.

In the next step we solve the equations (2) in the regime of small density perturbations, $\delta\rho_a/\rho_a \lesssim 1$. More specifically, we take a sub-Jeans regime for baryons, that are forced inside an already formed dark matter halo by its gravitational acceleration \mathbf{g} . For a realistic choice of parameters, the scattering cross sections are large, leading to $1/\tau \rightarrow \infty$ as good zeroth order approximation, in which case all velocity differences are nil. Thus, assuming that initial distribution of elements is uniform, one gets a relation between gradients of individual pressure contributions and \mathbf{g} (see, e.g. Chuzhoy & Nusser 2003),

$$\frac{\nabla P_a}{n_a m_a} = \frac{\sum n_a m_a}{m_a \sum n_a} \mathbf{g} = \frac{\bar{m}}{m_a} \mathbf{g}, \quad (4)$$

and the quasi-static version of Eq. (2) reduces to a set of algebraic equations,

$$\mathbf{g} \left(1 - \frac{\bar{m}}{m_a} \right) + \frac{q_a}{m_a} \mathbf{E} - \sum_b \frac{\mathbf{V}_a - \mathbf{V}_b}{\tau_{ab}} = 0. \quad (5)$$

We assume a quater mass fraction of ${}^4\text{He}$ so that $n_{{}^4\text{He}}/n_{\text{H}} = 1/12$ and

$$\bar{m} = \frac{4m_p + 12m_p}{1 + 12} = \frac{16}{13}m_p. \quad (6)$$

Notice that small reduction of \bar{m} due to ionized fraction can be safely neglected.

For the two dominant neutral components, hydrogen and helium, the solution is readily found

$$\frac{\mathbf{V}_{\text{He}} - \mathbf{V}_{\text{H}}}{\tau_{{}^4\text{HeH}}} = \mathbf{g} \left(1 - \frac{\bar{m}}{m_{{}^4\text{He}}} \right) = \frac{9}{13} \mathbf{g}. \quad (7)$$

One can immediately see that quite naturally, $\mathbf{V}_{\text{He}} - \mathbf{V}_{\text{H}} \parallel \mathbf{g}$.

We now turn to the diffusion of charged particles, which have an additional complication due to \mathbf{E} . Solving the equation for electrons, and using large disparity between m_e and any nuclear mass, one can easily get that

$$\mathbf{E} = -\frac{\bar{m}}{e} \mathbf{g}, \quad (8)$$

where e is the positron charge. Carrying this to the equation for protons (or H^+), we get the following solution for the relative diffusion velocity:

$$\begin{aligned} & (\mathbf{V}_p - \mathbf{V}_{\text{H}}) \times \left(\frac{1}{\tau_{\text{pH}}} + \frac{1}{\tau_{\text{pHe}}} \right) \\ &= -\mathbf{g} \left(\frac{2\bar{m}}{m_p} - 1 - \frac{\tau_{\text{HeH}}}{\tau_{\text{pHe}}} \right) = -\mathbf{g} \left(\frac{19}{13} - \frac{\tau_{\text{HeH}}}{\tau_{\text{pHe}}} \right). \end{aligned} \quad (9)$$

If one decides to neglect the helium contribution, $n_{\text{He}}/n_{\text{H}} \rightarrow 0$, then $\tau_{\text{pHe}}^{-1} \rightarrow 0$, $\bar{m} \rightarrow m_p$ and we get

$$\frac{\mathbf{V}_p - \mathbf{V}_{\text{H}}}{\tau_{\text{pH}}} = -\mathbf{g} \left(\frac{\bar{m}}{m_p/2} - 1 \right) \rightarrow -\mathbf{g}. \quad (10)$$

This equation is especially easy to interpret: the effect of the EM force is such that the motion of e and p is tightly coupled together, so that their effective mass per particle is $m_p/2$, and indeed lighter than \bar{m} . This results in the diffusion of both e and H^+ *against* the direction of the gravitational acceleration.

We are now ready to include the diffusion of Lithium, using already found solutions for

$\mathbf{V}_p - \mathbf{V}_{\text{H}}$ and $\mathbf{V}_{\text{He}} - \mathbf{V}_{\text{H}}$. The general expression is given by

$$\begin{aligned} & (\mathbf{V}_{\text{Li}} - \mathbf{V}_{\text{H}}) \left(\frac{1}{\tau_{\text{LiH}}} + \frac{1}{\tau_{\text{LiHe}}} + \frac{1}{\tau_{\text{Lip}}} \right) \\ &= \mathbf{g} \left(\frac{2\bar{m}}{m_{\text{Li}}} - 1 + \left[1 - \frac{\bar{m}}{m_{\text{He}}} \right] \frac{\tau_{\text{HeH}}}{\tau_{\text{LiH}}} \right. \\ & \left. - \left[\frac{2\bar{m}}{m_p} - 1 - \frac{\tau_{\text{HeH}}}{\tau_{\text{pHe}}} \right] \frac{\tau_{\text{pH}}\tau_{\text{pHe}}}{\tau_{\text{Lip}}(\tau_{\text{pH}} + \tau_{\text{pHe}})} \right), \end{aligned} \quad (11)$$

and the $n_{\text{He}}/n_{\text{H}} \rightarrow 0$ limit of this rather cumbersome formula reduces to the following simplified expression:

$$(\mathbf{V}_{\text{Li}} - \mathbf{V}_{\text{H}}) \left(\frac{1}{\tau_{\text{LiH}}} + \frac{1}{\tau_{\text{Lip}}} \right) = \mathbf{g} \left(\frac{5}{7} - \frac{\tau_{\text{pH}}}{\tau_{\text{Lip}}} \right). \quad (12)$$

In the last formula, we approximated $m_{\text{Li}} = 7m_p$. The direction of the lithium diffusional velocity is far from obvious: it depends on the competition of the two terms on the r.h.s. of (12), and *if* the friction relative to p wins (*i.e.* small τ_{Lip} limit), the motion of Li^+ ions will trace the motion of ionized fraction of hydrogen gas. Indeed, Eq. (12) reduces to $(\mathbf{V}_{\text{Li}} - \mathbf{V}_{\text{H}})/\tau_{\text{pH}} = -\mathbf{g}$, or $\mathbf{V}_{\text{Li}} = \mathbf{V}_p$, if τ_{Lip}^{-1} is the largest parameter. We now need additional information about the size of diffusional coefficients τ_{ab}^{-1} .

The scattering of ${}^4\text{He}$ on p has been calculated numerically by Chung & Dalgarno (2002). The value of the transport cross section is close to $\sigma_{\text{HeH}} \simeq 100a_B^2$ in the range of energies we are interested in, where $a_B = 1/(am_e)$, and the average over the Maxwellian velocity distribution is then

$$\langle \sigma_{\text{HeH}} v^3 \rangle \simeq 6 \left(\frac{T}{\mu_{14}} \right)^{3/2} \sigma_{\text{HeH}}, \quad (13)$$

where T is the temperature of the baryonic fluid, and $\mu_{14} = 4m_p/5$.

The cross sections of a singly charged ion on a neutral atom can be approximated by the following formula (Mott & Massey 1934):

$$\sigma_{ab} \simeq 2.2\pi \left(\frac{\alpha_{\text{pol}}(b)\alpha}{E} \right)^{1/2}, \quad (14)$$

where $\alpha_{\text{pol}}(b)$ is the atomic polarizability of the neutral species b : $\alpha_{\text{pol}}(\text{H}) = \frac{9}{2}a_B^3$ and

$\alpha_{\text{pol}}(\text{He}) = 1.38a_B^3$. It has to be noticed that p -H scattering is in practice a more complicated process due to the identical nature of the nuclei involved. Far more elaborate treatment of the p H cross section can be found in Glassgold et al. (2005). For the accuracy of our discussion, we shall still approximate it with (14), and the Maxwellian average that enters in the diffusion coefficients is given by

$$\langle \sigma_{ab} v^3 \rangle \approx 20\pi a_B^2 \frac{TRy^{1/2}}{\mu_{ab}^{3/2}} \left(\frac{\alpha_{\text{pol}}(b)}{\alpha_{\text{pol}}(\text{H})} \right)^{1/2}, \quad (15)$$

where Ry stands for the hydrogen binding energy, $\alpha^2 m_e / 2 \approx 13.6$ eV. This is used to calculate τ_{LiH} , τ_{LiHe} , and with the caveat pointed above, τ_{pH} . Finally, and most importantly, the p Li scattering is given by the Rutherford formula,

$$\langle \sigma_{\text{Li}p} v^3 \rangle = \frac{8\pi^{1/2} \alpha^2}{(2T)^{1/2} \mu_{17}^{3/2}} \times \ln \Lambda, \quad (16)$$

where $\mu_{17} = \frac{7}{8} m_p$, and $\ln \Lambda$ is the so-called Coulomb logarithm. For the conditions of primordial plasma after the recombination, its value is large, $\ln \Lambda \sim 40$, and weakly dependent on temperature. Because of the long-range nature of the EM force, Eq. (16) exhibits strong enhancement at small velocities/low temperatures.

Finally, the abundance of free protons in the primordial plasma is connected to a well-known X_e value, the electron ionization fraction,

$$\frac{n_p}{n_H}(T) \approx \frac{9}{8} X_e(T), \quad (17)$$

which is of course an explicit function of temperature (or redshift). We are now ready to determine the sign of the r.h.s. bracket in the simplified formula (12):

$$\begin{aligned} \frac{5}{7} - \frac{\tau_{p-H}}{\tau_{\text{Li-p}}} &= \frac{5}{7} - \frac{n_p}{n_H} \frac{\mu_{17}^2 m_p}{\mu_{11}^2 m_{\text{Li}}} \frac{\langle \sigma_{\text{Li}p} v^3 \rangle}{\langle \sigma_{\text{pH}} v^3 \rangle} \\ &\sim \frac{5}{7} - 300 \times \frac{X_e}{10^{-3}} \times \frac{\ln \Lambda}{40} \times \left(\frac{0.01 \text{ eV}}{T_{\text{baryons}}} \right)^{3/2}. \end{aligned} \quad (18)$$

It is easy to see that for the cosmological parameters between recombination and reionization, the expression (18) is negative. In

Figure 2, we plot the separatrix for the positivity of this expression on X_e -redshift plane, assuming standard relations between T , photon temperature T_γ and redshift z .

One can see that even tiny values of X_e would lead to an outward diffusion of lithium. The separatrix stays firmly below cosmological $X_e(T)$ at all redshifts, and therefore one can conclude that the direction of lithium diffusion is anti-parallel to the local gravitational force,

$$\frac{5}{7} - \frac{\tau_{\text{pH}}}{\tau_{\text{Li}p}} < 0 \implies \mathbf{V}_{\text{Li}} - \mathbf{V}_{\text{H}} \propto -\mathbf{g}. \quad (19)$$

One should take a notice that inclusion of He into this analysis does change the conclusions somewhat: while lithium remains tightly bound to H^+ , the outward diffusion of ionized H^+ is no longer guaranteed. We find, however, that in most of the redshift window of interest, $z > 30$, both p and Li diffuse out of overdensities. Therefore, the qualitative details of this simplified analysis are not going to change.

3. Discussion: do we understand the magnitude and sign of possible $[\text{Li}/\text{H}]$ variations?

While we have shown that, quite unexpectedly, the direction of lithium diffusion in the early Universe after the recombination is against local gravitational force, it is far less clear if this has any implications for the lithium problem at $O(1)$ level. Indeed, so far we have used small baryonic perturbations, $\delta\rho/\rho \ll 1$, so that the effect of diffusion on any element is not going to be large by definition. While the analysis of the nonlinear case appears to be extremely challenging, and certainly falls outside the scope of the present work, we could provide simple estimates of the possible size of the effect due to gravity, and potentially due to other forces that act differentially on charged and neutral particles.

Let us assume to good accuracy that $1/\tau_{\text{Li}p}$ is the largest coefficient, so that lithium motion is linked to the motion of ionized H^+ . Then, using the continuity equations we can tie the variation in lithium abundance that develops at

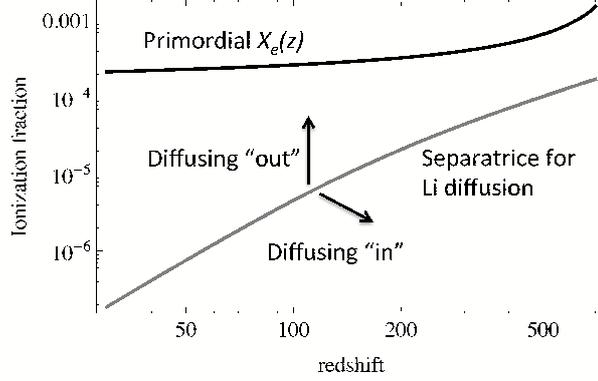


Fig. 2. Black curve is the standard post-recombination ionization fraction $X_e(T)$. Gray curve is the separatrice for $\mathbf{V}_{\text{Li}} - \mathbf{V}_{\text{H}}$ being parallel or antiparallel to \mathbf{g} . Above the gray line Eq. (19) holds, and since it stays always below the black curve, lithium diffuses “out”, leading to $[\text{Li}/\text{H}]_{\text{grav min}} < [\text{Li}/\text{H}]_{\text{SBBN}}$.

time t_f to the halo density,

$$\begin{aligned} \frac{\Delta[\text{Li}/\text{H}]}{[\text{Li}/\text{H}]_{\text{SBBN}}} &\simeq \frac{\Delta p/\text{H}}{p/\text{H}} \simeq \int_{t_i}^{t_f} dt \frac{\nabla \mathbf{g}}{1/\tau_{p\text{H}}} \\ &= \int_{t_i}^{t_f} dt 4\pi G_N \rho_{\text{total}} \tau_{p\text{H}} \\ &= \frac{3}{2} \int_{t_i}^{t_f} dt \frac{\rho_{\text{total}}}{\bar{\rho}} \left(\frac{\dot{a}}{a}\right)^2 \tau_{p\text{H}}. \end{aligned} \quad (20)$$

Here \bar{p}/H is the average ionized fraction of hydrogen in the Universe, $\bar{\rho}$ is the average matter density, while ρ_{total} is the mass density of the halo, which is presumably contributed to mostly by the cold dark matter. The Hubble expansion rate, $\text{Hubble} \equiv \dot{a}/a$, makes appearance in this formula. Notice that while $\bar{\rho}$, Hubble, $\tau_{p\text{H}}$ depend on time/redshift in a simple calculable way, ρ_{total} is a highly complicated quantity that depends on many factors. Most importantly it is going to fluctuate from halo-to-halo, and can be widely different. The naive estimates of the parameters here give variation of lithium abundance at

$$\frac{\Delta[\text{Li}/\text{H}]}{[\text{Li}/\text{H}]_{\text{SBBN}}} \sim O(10^{-4}) \times \rho_{\text{total}}/\bar{\rho} \sim 10^{-2} \quad (21)$$

level, if we assume typical overdensity on the order of hundred times larger than average cosmological ρ .

Therefore, judging by these estimates, one would assume that the diffusion-induced variation of lithium abundance during the evolution after recombination is going to be small. That does not mean, however, that such variations will be small *everywhere* in the Universe. Therefore, it appears possible to find some isolated halos that could provide significant overdensities, and thus create $O(1)$ effect on lithium abundance in a spacially small patch. Therefore, in light of our findings of the physical mechanisms that can lead to the depletion of lithium in overdensities, we would like to re-state lithium problem in the following way: Instead of asking “how lithium got destroyed?”, one can ask “how likely that we live in the special part of the Universe, where lithium was depleted even before stars were born?”.

Finally, we would like to stress that perhaps the most significant part of our finding is that lithium remains closely tied to ionized fraction of the gas. Therefore, any *additional* forces \mathbf{f}_a that act on neutral and charged components differentially, can create variations in lithium abundance. Interesting candidates for creating such forces are radiation pressure/stellar winds from first stars, and possibly primordial magnetic fields. The whole scope of issues raised

by this work is still largely unexplored, but certainly deserves close attention due to the continuing interest in lithium problem.

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