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Roger Joseph Boscovich between geometry and astronomy

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Abstract. A short account of Boscovich's work on determination of the cometary orbits on the basis of three observations is given

1. Introduction

At the age of 14, Roger Joseph Boscovich moved from Dubrovnik to Rome, where he began his studies first in S. Andrea delle Fratte, and later at the *Collegium Romanum*, the Jesuits school. He was so brilliant in Mathematics and Physics, that in 1740 he was appointed professor of mathematics in the college. Thus, the beginning of his studies coincides with the first developments of differential calculus. However his mathematical knowledge remained essentially oriented towards the geometrical methods of, e.g., Newton's *Principia*, that he was very familiar with. Most of Boscovich's mathematical activity has been devoted to problems in Physics and Astronomy. The present note contains a short account of some geometrical problems in Astronomy that Boscovich has investigated. Particular attention is paid to his work on cometary orbits.

At the beginning of the XVIII century astronomers and mathematicians were concerned with two main problems, namely the calculation of the orbit of an observed comet and the deviations of Saturn and Jupiter from the Keplerian elliptic orbits. The discovery of Uranus in 1781 added a third problem, namely the determination of the orbit of the new planet.

The common background of these three problems was Newton's theory of planetary motions, as exposed in *Principia* Newton (1687). The main question was whether Newton's new theory of gravitation could explain the observed phenomena.

The inequalities in the motion of Jupiter and Saturn had been noticed already by Kepler. The common knowledge at the beginning of the XVIII century was mainly based on a few letters by Kepler, where he wrote that Jupiter appeared to accelerate and Saturn to decelerate with respect to the predicted elliptic motion. Actually, Kepler had written a preliminary unpublished note that has been found among his manuscripts and has been included in his *Opera Omnia* (Kepler, Frisch 1866, see Giorgilli 2012 for a report on Kepler's note). In view of the accumulation of astronomical data confirming the strange behaviour of the two biggest planets the Académie Royale des Sciences de Paris proposed three prizes in 1748, 1750 and 1752. Two of them (1748 and 1752) were awarded to Euler, while the 1750 one was not assigned. Boscovich presented a note for the 1752 prize that was

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considered as worth of publication in the Académie's *Memoires*. However, due to long delays, Boscovich eventually decided to publish it independently (Boscovich 1756).

In 1781 Frederick William Herschel announced the discovery of a new celestial object, that he at first thought to be a new comet. Actually, it had been already observed and catalogued as a fixed star by John Flamsteed in 1690, and later also by Pierre Lemonnier between 1750 and 1769 and by Christian Mayer in 1759, who however did not recognize it as a planet. Boscovich took an active part in furnishing a theoretical support for the calculation of the orbit, on request by Jerome de Lalande and Bouchart de Saron. He first tried to apply his method for determining the cometary orbit. However, after a few attempts he realized that according to his calculations the radial component of the velocity of the planet seemed to remain very small for a significant part of the orbit, which suggested that the new object could be moving on a circle. Meanwhile, similar calculations, but using the new established analytical methods, had been performed, among others, by Anders Johan Lexell (1783). Boscovich's work was still written in the traditional geometric language, which likely made his contributions to remain essentially underestimated.

I come now to the problem of cometary orbits, that will be the main subject of the present note. Newton's gravitational theory had opened a way towards the understanding of the comet's nature, a problem that had puzzled astronomers for many centuries. Prop. XL, Theor. XXI in Liber III of Newton's *Principia* states:[1]

> Cometas in sectionibus conicis umbilicos in centro Solis habentibus moveri, & radiis ad Solem ductis areas temporibus proportionales describere.

That is: the comets are just celestial objects like the planets or the satellites. A few pages later, with Prop. XLI Prob. XX, Newton raises the following problem, that he recognizes as *longe difficillimum* (exceedingly difficult):[2]

> Cometæ in Parabola moventis Trajectoriam ex datis tribus observationibus determinare.

We may understand both the relevance and the difficulty of the problem on the basis of the following considerations. A comet becomes visible only when it is sufficiently close to the Sun – at least it was so in Newton's epoch and later, with the available instruments. Furthermore, is was almost impossible to collect regular observations, because after a short period the comet could become unobservable, due, e.g., to a conjunction with the Sun or to a cloudy sky. This was a major obstacle in following the comet's path on the sky and in recovering it after a period of non observability, e.g., when a comet after passing behind the Sun would reemerge on the other side.[3] Thus, being able to calculate at least an approximated orbit as soon as possible was essential for astronomers.

Theoretically, three observations should provide enough information in order to calculate a good approximation of the orbit. As a matter of fact, this is just theory. In rough terms, we can observe only *angles*, e.g., latitude and longitude, the distance of the object from the Earth remaining unknown. The velocity could be calculated only by using observational data close enough so that the trajectory could be approximated with a straight segment, and assuming a constant speed, thus introducing a geometric approximation. On the other hand, the unavoidable observational error went amplified by the calculation, possibly producing totally unreliable results. The mathematical tools available in Newton's and Boscovich epoch were not strong enough in order to give a satisfactory solution to these problems. However, Boscovich has been able to find an useful recipe.

[1] "The comets move on conic sections having their umbilici on the Sun, and with the radii drawn to the Sun sweep areas proportional to times."

[2] "To determine the trajectory of a comet moving on a parabola from three given observations."

[3] We should also remark that at that time an effective collaboration between astronomers at different places was very difficult to establish, due to the long time needed in order to exchange the observational data.

Boscovich published four main texts on the subject. The first one is the inaugural thesis for the academic year 1746 at the *Collegium Romanum* (Boscovich 1746, see Tagliaferri & Tucci 1988 for a recent exposition and references). Two memories, containing major improvements with respect to the first one, were submitted to the *Académie des Sciences* of Paris while Boscovich was still a professor in Milano Boscovich (1774a) and Boscovich (1774b). In 1773, after the suppression of Jesuits order, he moved to Paris, where his memories were printed in 1774. Ten years later, after returning to Italy in Bassano, he gave a complete and detailed exposition of his method in the third volume of his *Opera pertinentia* Boscovich (1785). The next section is based in particular on the two memories Boscovich (1774a) and Boscovich (1774b) and on Volume III of Boscovich (1785).

2. Determining the orbit of a comet from three observations

My aim in this section is to discuss in some detail the main technical points that make the method of Boscovich interesting and practically useful. I will not try to give a complete account, which would exceed the limits of the present note.

The basis of the method is found in Newton's *Principia*. Newton assumes that the orbit of the comet is a parabola, i.e., the eccentricity is 1. This may appear unjustified in view of our current knowledge. However, it is enough to draw the part of the orbit close to the perihelion in order to get convinced that if the eccentricity is, say, between 0.95 (elongated ellipse) and 1.05 (hyperbola) then the curves get practically superimposed.[4] On the other hand, assuming the eccentricity to be 1 means that one of the parameters of the ellipse is known, which considerably simplifies the problem. The method of Newton had been reelaborated by Cassini (1729) and Bouguer (1735).

The drawback of Newton's method is that it relies on approximating the comet's orbit, for a short time interval (a few days), with a straight line, and assuming that the velocity is uniform. This often leads to wrong results, which likely occur in practical cases (see Tagliaferri & Tucci 1988). Boscovich introduces some clever changes that provide a better approximation and allow one to use observations over longer time intervals, thus reducing the influence of the observational error on the results. Moreover, he also emphasizes that his method turns out to be quite effective, being based essentially on a few geometrical figures and on the use of the *circinus proportionis* in order to speed up the calculation.

The data to be submitted to calculation are the usual ones. One needs three observations, over a not too long time interval (a few days, that may be extended up to 10, 20 or 30 days, thanks to Boscovich's improvements). The observations give the right ascension and the declination of the comet as seen from the Earth, and the first operation, which is just a geometric one, is the projection on the ecliptic's plane, thus getting the longitude of the comet at three given times. Meanwhile, using astronomical tables, one gets the longitude of the Sun at the time of the observation.

The problem is to calculate the elements of the orbit (the inclination, the longitude of the node, the longitude of the perihelion and the parameter of the parabola, the eccentricity being fixed to be 1) and the motion along the orbit, so that the position of the comet on the sky may be predicted.

The calculation of the inclination and of the longitude of the node is a matter that is solved by geometrical methods. Essentially it is matter of finding the intersection between the plane of the ecliptic and the plane passing through the Sun that the comet's orbit belongs to, which in turn can be determined by the observations. I skip this part of the method, which of course is a relevant one but is a standard matter in astronomy. I shall rather refer to *reduced* quantities, namely quantities related to the projection of the comet's positions in the ecliptic plane.

[4] By the way, the most famous comet, namely Halley's, has an eccentricity about 0.967 .

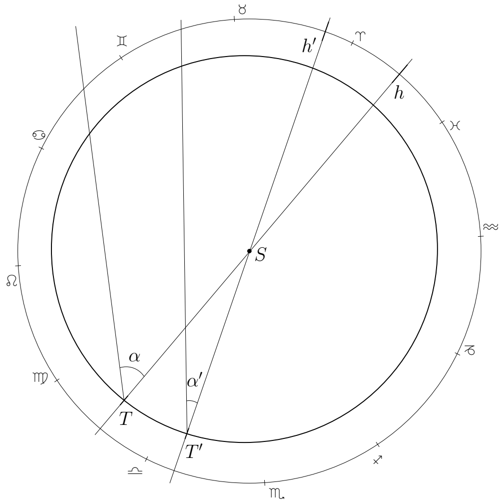

Fig. 1. Calculation of the position of the comet in the heliocentric system. The relative longitude θ with respect to the Sun and the distance c between the two position of the comet are determined as functions of the distances δ and δ' from the Earth.

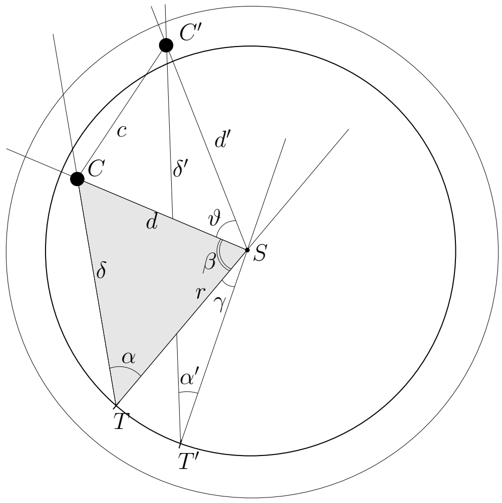

Fig. 2. Using the second observation and the uniformity of the motion on the chord the distance δ' can be calculated as a function of δ.

2.1. Geometrical setting

Let us collect three observations at times $t < t'' < t'$, so that the longitudes of the comet at these three times are known. The first problem is to determine the position of the comet in a heliocentric system using the observations from the Earth. Boscovich's suggestion is to represent the data for the first and third observation as in figure 1. The orbit of the Earth is represented as a circle[5] with radius, say, 1. A larger circle represents the ecliptic (radius 1.2 suggested), with center displaced by 0.017 units in the direction opposite to the perihelion of the Earth's orbit, 9°Capricorn at Boscovich's time. The zodiac signs may be marked on the ecliptic, which helps drawing the rest of the figure. Let t and t' be the epochs of the first and of the third observation, and let h and h' be the corresponding longitudes of the Sun on the ecliptic, as computed from the tables. Then the Earth is located at the points T and T', and by calculating the differences α and α' between the longitudes of the comet and of the Sun the direction of observation of the comet with respect to the Earth is drawn.

Assume now that at time t (the first observation) the comet is at distance δ from the Earth, so that the expected position C of the comet is given (Fig. 2). This is rather arbitrary, since the distance can not be observed, but Boscovich gives some hints how to guess it not too different from the true value. Thus the triangle STC with vertices in Sun, Earth and the comet can be completely determined, since δ, r and α are known. This gives the angle β between the Earth and the comet as seen from the Sun and the distance d of the comet from the Sun as functions of δ. Similarly, picking an arbitrary value for the distance δ' at the epoch of the third observation, the triangle $ST'C'$ can be completely determined, thus giving $\beta' = \gamma + \beta + \theta$ and d' as functions of δ'. Finally, since the angle γ is known, the angle θ between the positions C and C' as seen from the Sun can be calculated, so that also the triangle SCC' is completely determined. This gives also the distance c between the two positions of the comet as function of δ and δ'. Thus, the complete configuration is given as a function of two arbitrary quantities.

5 The eccentricity of the Earth is 0.0168, which makes the ellipticity to become practically invisible on a figure

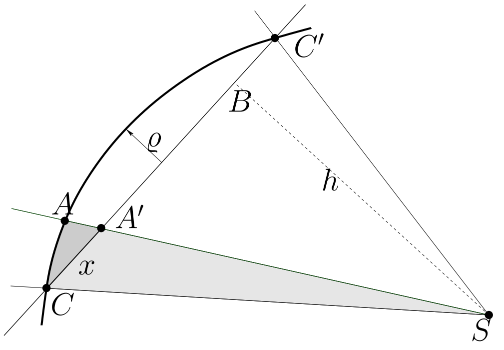

Fig. 3. By applying the second Kepler's law one sees that the motion on the chord is approximately uniform (see text).

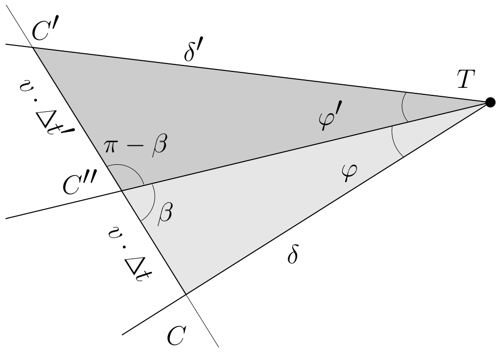

Fig. 4. Using the second observation and the uniformity of the motion on the chord the distance δ' can be calculated as a function of δ.

2.2. Dynamical considerations

By dynamical considerations one obtains three relevant properties that characterize the motion in the gravitational field.

– (i) During the motion on a short segment of a conic section, the velocity on the arc needs not be uniform, but if one considers the motion along the chord then the velocity is approximately constant (see figure 3).
– (ii) If two masses move on circular orbits with radii r and r', respectively, then the corresponding velocities $\mathbf{v}$ and $\mathbf{v}'$ satisfy $r|\mathbf{v}|^2 = \mathbf{r}'|\mathbf{v}'|^2$, i.e., the velocity is proportional to the inverse of the square root of the radius.
– (iii) Let two bodies be at the same distance r from the Sun, and move on a circle with velocity parabola with velocity

Property (i) is one of the main remarks of Boscovich. The claim is easily checked by looking at figure 3 and recalling the second Kepler's law. For if ρ is small then the area of the small curvilinear triangle CAA' is of order ρ, and may be neglected. On the other hand, the area of the triangle SCA' is $xh/2$, i.e., it is proportional to the distance $|CA'|$, and by the second Kepler's law it is approximately proportional to time. Thus, x is approximately proportional to time.

Property (ii) is nothing but the third Kepler's law applied to circular orbits.

Property (iii) is a theorem by Newton (*Principia*, Prop. XVI theor. VIII, Corol. 7 of Liber I) that Boscovich refers to without proof. However, it is easy for us to check the statement by using conservation of energy (that Newton did not use). For, the energy is $E = |\mathbf{v}|^2/2 - k/r$ with a constant k and the parabola corresponds to $E = 0$, so that $\left|\mathbf{v}_{\rm par}\right|^2 = 2k/r$. On the other hand on a circular orbit the centrifugal force must compensate exactly the gravitational attraction, which means that $\left|\mathbf{v}_{\rm cir}\right|^2/r = k/r^2$, i.e., $\left|\mathbf{v}_{\rm cir}\right|^2 = k/r$. The claim follows.

2.3. Calculating the distances

We may now tackle the problem of calculating the distances. This is made in two steps. The first one is to find a way to calculate δ' as a function of δ, so that one free parameter is left. Here the second observation enters the game together with property (i) of the previous section. The second problem is to find an equation for δ, so that it can be determined and, as a consequence, the projection of the parabola on the ecliptic plane may be found. Here the improvement between the first memoir Boscovich (1774a) and the second one Boscovich (1774b) is found, since Boscovich introduces a correction that allows one to increase the time distance between observations.

Let us first consider the calculation of the distance δ' for the third observation as depending on δ. Here property (i) and the second observation are used. Let us consider a geocentric system, as in figure 4, and let C and C' be the position of the comet for the first and third observation, at times t and t', respectively. The direction of the second observation, at time t'', intersects the segment CC' at some point C'', and the angles ϕ, ϕ' are known. In view of property (i) the motion on the segment CC' is approximately uniform,[6] and so the segments CC'' and $C''C'$ have length $v \cdot \Delta t$ and $v \cdot \Delta t'$ respectively, where v is the constant velocity, $\Delta t = t'' - t$ is the interval between the first and the second observation and $\Delta t' = t' - t''$ is the interval between the second and the third observation. By the law of sines we get the relations

$$\frac{\sin\beta}{\delta} = \frac{\sin\phi}{v\Delta t} \qquad \text{for the triangle } CTC'' \ ,$$
$$\frac{\sin(\pi-\beta)}{\delta'} = \frac{\sin\phi'}{v\Delta t'} \qquad \text{for the triangle } C''TC' \ .$$

Thus one readily gets the relation

$$\frac{\delta}{\delta'} = \frac{\Delta t}{\Delta t'} \cdot \frac{\sin\phi'}{\sin\phi} \ ,$$

which allows us to calculate δ' as a function of δ, so that only one free parameter is left (Boscovich 1774b, Theorema IV).

Now comes the most difficult part, namely how to get an equation for δ. A first approximation is found as follows. Using properties (ii) and (iii) of the previous section one readily gets the relation $rv^2 = 2r_T v_T^2$, where r, r_T are the distances of the comet and of the Earth, respectively, from the Sun, and v, v_T are the velocities. The radius and the velocity of the Earth are known. The velocity of the comet may be calculated as $v = \frac{c}{t'-t}$, where $c = |CC'|$ is the distance between the observations at times t, t'. The radius for the comet may be taken as the average between the radii d, d' at the observed points C, C'. Thus, in Boscovich's notations we get the relation

$$bc^2 = a \ , \quad b = d + d' \quad , \ a = 2r_T v_T^2 (t'-t)^2 \ ,$$

where the velocity has been replaced by the space. Recalling that c, d, d' are all calculated as functions of the unknown distance δ of the first observation from the Earth the l.h.s. turns out to be a function of δ, while the r.h.s. is a constant. The problem is then reduced to the solution of an equation, that can be looked for using the *regula falsi* (actually a scheme of successive approximations). Boscovich emphasizes that by making a first guess with graphical methods (that he describes in great detail) the number of operations to be actually performed can be made reasonably small: typically, he says, the result is found after three or four approximation steps.

In the second memoir Boscovich (1774b), § 12 Boscovich explains how to look for a better approximation of the distance, which he claims to be a major improvement. Thus he corrects the formula above as

$$bc^2 - \frac{c^4}{12b} = a \ .$$

Let me sketch the argument of Boscovich making reference to figure 5 (see caption for the construction and some relevant properties). The basic idea comes from the remark that the actual velocity on the chord CC' is not really constant. Boscovich's suggestion is to replace the distance $b/2$ in his first formula with the distance of a point on the parabola having an average velocity, that he wants to determine (Boscovich 1774b, § 12). To this end, let us translate the chord CC' so that it becomes tangent to the parabola in D. This is very easy if the points CC' are taken symmetrically with respect to the axis of the parabola, as in figure 5, since the point D actually coincides with the vertex of the parabola, but a similar construction is easily made in any case.

Let the point O in figure 5 be chosen so that $3|DO| = |DF|$, and LL' be the line through O parallel to the chord CC'. The key remark is

[6] The motion is approximately uniform on the chord of the parabola. Here the points are translated due to the reduction to a geocentric system, but Boscovich proves that replacing both the Earth and the comet's motion on the orbit with the motion on the corresponding chords then the velocity of the comet as seen from the Earth remains approximately uniform on the chord (Boscovich (1774b), Theorema III).

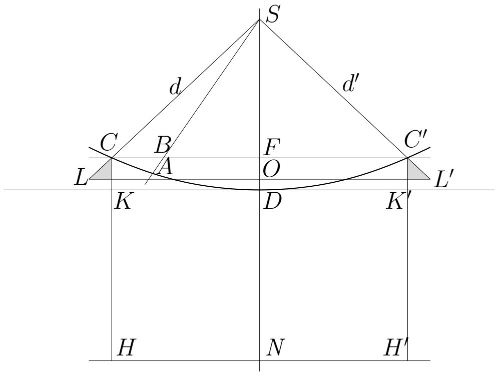

Fig. 5. Illustrating the search for an average velocity in the simplest case. S is the focus (the Sun), the straight line HH' is the directrix and D is the vertex (the perihelion). The positions C, C' are chosen to be symmetric with respect to the axis of the parabola, which makes the geometric considerations simpler. Thus, the tangent to the parabola in D is parallel to the chord CC'. By the properties of the parabola the equalities $|SD| = |DN|$, $|SC| = |CH|$ and $|SC'| = |C'H'|$ hold true. The point F is the intersection of the chord CC' with the radius SD, and the point O is such that $|DF| = 3|DO|$. Recalling that the area between the parabola CDC' and the straight line KDK' is $\frac{1}{3}$ of the area of the rectangle $CC'K'K$ we get that the area of the triangle LSL' differs from the area of the sector $CDC'S$ of parabola only by the two small grey triangles with vertices in L, L', which may be neglected if $|DF|$ is small.

that the area of the triangle $LSL'S$ is approximately the same as the area of the sector of parabola CSC' swept by the radius joining the comet with the Sun (see caption for figure 5). On the other hand, also the motion along the segment LL' is approximately uniform, by the same argument used for the chord.[7] Denote by $\overline{v}$ the average velocity along LL' and by v_O the actual velocity in O, so that we have $v_O \simeq \overline{v}$. Now comes a short calculation. In view of Thales theorem we have $\frac{v_O}{v_D} = \frac{r_O}{r_D}$, where v_D is the actual velocity in D and the short notations $r_O = |SO|$ and $r_D = |SD|$ have been introduced. Remark that the equality applies only to the point D, for only there the velocities in O and D are parallel. Recall also that in view of the second Kepler's law at any point A of the parabola one has $r_A v_A^2 = r_D v_D^2$, with a similar meaning of the symbols as above. Look now for a point on the parabola such that its velocity equals the average one; that is, its distance r from the Sun obeys $r\overline{v}^2 = r_D v_D^2$. Thus, in view of $v_O \simeq \overline{v}$ and of the application above of Thales theorem, both the (approximate) equalities

$$\frac{\overline{v}^2}{v_D^2} = \frac{r_D}{r} \quad \text{and} \quad \frac{\overline{v}^2}{v_D^2} = \frac{r_O^2}{r_D^2}$$

must be satisfied, and by setting $x = \frac{1}{3}|DF|$, which is a small quantity, we get

$$\begin{aligned} r &= \frac{r_D^3}{r_O^2} \\ &= \frac{r_D^3}{(r_D - x)^2} \\ &= \frac{r_D^3}{r_D^2 - 2r_D x + x^2} \simeq r_D\left(1 + \frac{2x}{r_D}\right) . \end{aligned}$$

This is the distance to be used in place of $b/2$.

Let us now express the distance r in terms of $b = d + d'$ and $c = |CC'|$. To this end remark that for the parabola we have $|DF| = \frac{c^2}{16r_D}$, which is a small quantity if the points C, C' are close. On the other hand we also have

$$\begin{aligned} \frac{b}{2} &= \frac{d + d'}{2} \\ &= \frac{|CH| + |C'H'|}{2} \\ &= r_D + |DF| = r_D + \frac{c^2}{16r_D} , \end{aligned}$$

which, keeping only the lowest order term in c, may be inverted as

$$r_D = \frac{b}{2} - \frac{c^2}{8b} .$$

[7] The argument here is somehow simplified with respect to the wide and detailed discussion that can be found in Boscovich (1785). For, Boscovich introduces the average velocity along the chord CC', which makes the calculation longer than the present one, although the latter is essentially equivalent to Boscovich's. A further simplification is that I consider the special case in which the points C, C' are placed symmetrically with respect to the axis of the parabola. However all the calculations apply, with minor changes, to the general case. It is just matter of properly playing with Thales theorem.

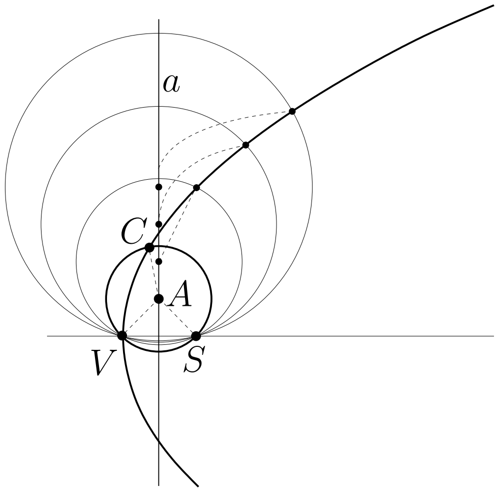

Fig. 6. The motion of the comet on a parabolic orbit.

Putting this in the expression above for r together with $2x = \frac{2}{3}|DF| = \frac{c^2}{12b}$ we finally get

$$r = \frac{b}{2} - \frac{c^2}{24b}\,.$$

This is the quantity to be substituted in place of $b/2$ in the first approximated formula $bc^2 = a$, so that the correction of Boscovich is found.

2.4. The motion on the orbit

Having determined the reduced distances (i.e., the projection of the position on the plane of the ecliptic) at the time of two observations the calculation of the orbit is almost a standard matter, if long and perhaps tedious. In brief, and using our language, the problem is to determine the perihelion ω and the parameter p of a parabola in a plane using the known position of the Sun, which is taken as the origin of a system of polar coordinates. This is easily made by inserting the known data r, θ from two observations in the well known equation of a parabola

$$r = \frac{p}{1 + \cos(\theta - \omega)}\,.$$

The last step is to determine the motion of the comet on the parabola, so that its position can be predicted. To this end Boscovich uses a geometric construction illustrated in figure 6 (Boscovich 1774b, Theorema VIII). Let S be the focus (i.e., the position of the Sun) and V be the vertex (the perihelion) of the parabola, and let a be the axis of the segment SV. Let A be any point of the axis a. Draw the circle with center in A and passing through the points S and V, and find the point C of intersection with the parabola. The claim is that *if the point A moves uniformly on the axis a then the point C represents the actual motion of the comet (i.e., it moves according to the second Kepler's law).* This has been proven by Newton (1687), in Prop. XXX Prob. XXII in Liber I.

3. After Boscovich

As I have said in section 2, Boscovich had a thorough knowledge of the Principia, and an excellent mastering of the geometric language of Newton. This is the language he uses, and sometimes he seems to be proud of his ability.[8] On the other hand, his contemporaries D'Alembert, Euler, Lagrange and Laplace, just to quote the most famous ones, had made major steps in developing the new methods based on differential calculus. This probably caused the work of Boscovich to be underestimated and openly criticized, if not despised.

The development of new analytical methods, definitely more powerful with respect to Boscovich's geometric ones, made probably more and more difficult to read his works. When he was writing his five volumes of *Opera Pertinentia*, in 1785, Euler, Lagrange and Laplace had already created the skeleton of perturbation theory. The problem of three bodies had already been investigated by Euler and Lagrange; the motions of the perihelia and of the nodes of the planetary orbits had

[8] The exposition of the corrected formula for the distances of the comet that I have reported above is fully worked out with geometric language. However, Boscovich ends the section by including also a short algebric calculation using the first term of a series expansion. Then he explains that he first wanted to do everything geometrically, and adds: *Mais j'aime beaucoup la Géometrie un peu trop méprisée, ou au moins négligée aujourd'hui.*

been explained by Lagrange, soon followed by Laplace; the first argument concerning the stability of the planetary orbits had been found by Lagrange; Laplace was discovering the *great inequality*, i.e., the long–period mutual perturbation between Jupiter and Saturn that eventually explained the strange behaviour noticed by Kepler one and half a century before; Lagrange was probably writing his first edition of the *Méchanique Analytique*, that was going to be published in 1788. All these exciting results were clearly obtained thanks to the new powerful tools offered by differential calculus and Analysis. To say it in a few words, Boscovich did not take part in the considerable effort of building Analysis.

At the beginning of the XIX century, about 15 years after Boscovich's death, a new puzzling problem was posed to astronomers and mathematicians. The discovery of Ceres on the 1'st of January 1801, due to Giuseppe Piazzi, raised the problem how to compute the orbit of the new object, which clearly was not a comet, and seemed to have been lost after passing behind the Sun. The problem was eventually solved by Gauss, who devised a method for approximating all elements of the orbit, without assuming it to be neither circular nor elliptic. The mathematical difficulty was definitely bigger than that of determining a parabola, but Analysis was developed enough, thanks also to the work of Gauss himself, in order to tackle it. Gauss could solve the problem, indeed, and his method, published in 1809, became the standard method for determining the orbits of all the new discovered celestial objects. We still use it, and Boscovich's method was soon completely forgotten.

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