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Solution of the NLTE line transfer problem by use of forth-and-back implicit Λ-iteration

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Abstract. The basic properties of the forth-and-back implicit Λ-iteration, a simple and fast convergent method developed to solve NLTE line transfer problem, are summarized.

Key words. Methods: numerical - Radiative transfer - Stars: atmospheres

1. Introduction

The basic difficulty in the solution of the NLTE line formation problem arises from the non-local and, in general, non-linear coupling between the radiation field and the excitation state of a gas. From the mathematical point of view the coupling is described by the simultaneous solution of the corresponding equations of radiative transfer and statistical equilibrium.

The most straightforward iterative method, the so–called Λ iteration, solves radiative transfer and statistical equilibrium equations in turn. However, in most cases of interest (in scattering dominated media of large optical thickness) the rate of convergence of this procedure is infinitely slow, in practice preventing the convergence to the exact solution.

The solutions currently in use are based either on linearization and on cumbersome matrix operations, or on the use of some physical (or computational) approximations of the Λ operator within an iterative procedure. The former ones are very time and memory consuming, whereas the latter ones, which usually require some free parameter controlling the convergence, almost always need additional acceleration by some mathematical techniques (Ng acceleration, overrelaxation method, etc.) to achieve high convergence rate (for a survey of different approaches and numerical methods see Atanacković - Vukmanović (2004)).

2. Forth-and-back implicit Λ iteration

In the last years we have developed (Atanacković - Vukmanović et al. 1997; Atanacković - Vukmanović 2003) a novel accurate and fast convergent iterative method, forth-and-back implicit Λ-iteration (FBILI), to solve the NLTE radiative transfer problems. The FBILI dramatically accelerates the convergence of the classical Λ iteration (while retaining its straightforwardness) by use of a forth-and-back approach together with an implicit representation of the source function in the computation of both the in-going and the out-going radiation field intensities. The values of the radiation field are unknown, however its propagation can be easily represented by using the integral form of the RT equation and assuming some polynomial (e.g.

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piecewise parabolic) representation of $S(\tau)$ between two successive depth points.

In the first part of each iteration (forward process), the FBILI differs from the classical Λ iteration which recalculates the in-going intensities $I^-_{\nu\mu}(\tau)$ from the old source function $S^o(\tau)$, in using $S^o(\tau)$ to compute and store, at each optical depth point $\tau_l (l = 1, N)$, the coefficients of the linear *local implicit* relation representing the values of the in-going intensities $I^-_{\nu\mu}(\tau_l)$ in terms of yet unknown values of the source function $S(\tau_l)$ and its derivative $S'(\tau_l)$. These coefficients are stored for further use in the backward process of computation of the new values of $S(\tau)$. The iterative computation of the coefficients of these *implicit* relations, rather than that of the intensities themselves, provides an extremely high convergence rate with respect to the classical Λ iteration scheme.

In the backward process one proceeds from the bottom (last layer) where $I^+_{\nu\mu}(\tau_N)$ or more precisely, the coefficients of the corresponding *implicit* relation for the out-going intensities are known. Together with the coefficients stored in the forward process for τ_N, it is easy to obtain the linear implicit relation between the total mean intensity of the radiation field and the source function which, substituted into the statistical equilibrium equation leads to the new value of $S(\tau_N)$. The new values of $S(\tau_N)$, $S'(\tau_N)$ and, hence, $I^+_{\nu\mu}(\tau_N)$ are then used to compute the coefficients of the linear relation in the next upper layer. The computation of the new source function together with the out-going intensities is performed during the backward process layer by layer to the surface.

Although the solutions are obtained for the general multi-level atom line formation problem, here we display excellent convergence properties of the FBILI method relative to, from the numerical point of view, the most difficult test case of the two-level atom line formation (with no overlapping continuum) in a constant property medium. Fig.1 displays the run with iterations of: (a) maximum relative correction between two successive iterations ($\delta = \max_l [2(S^i - S^{i-1})/(S^i + S^{i-1})]$), (b) average relative correction and (c) the residual error $\sqrt{\frac{1}{N}\sum_{l=1}^{N}(\frac{S^i - S^\infty}{S^\infty})^2}$, where S^∞ is the converged solution. The exact solution is obtained with $\delta = 10^{-3}$ in a very small number of iterations (of the order of 10) for arbitrarily small values of the NLTE parameter ε.

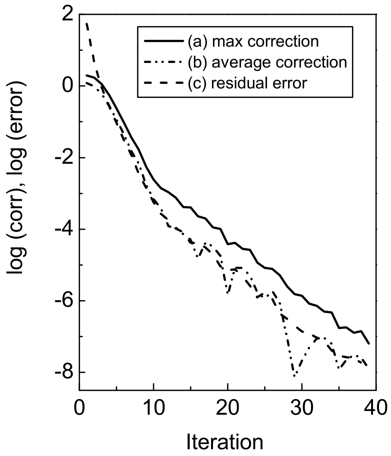

Fig. 1. Convergence properties of the FBILI method, displayed for the case $\varepsilon = 10^{-8}$.

The forth-and-back implicit Λ iteration is a simple, accurate and fast convergent method. The stability of the procedure is very high owing to the numerical operations that mimic the physical process of radiative transfer. The fact that no matrix operation is required and that the memory storage grows only linearly with the dimension of the problem makes this method very useful for more complex radiative transfer problems.

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