Mem. S.A.It. Suppl. Vol. 9, 71

Memorie della Supplementi

Solar dynamo models driven by a multi-cell meridional circulation

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Abstract. The effect of a two-cell meridional circulation pattern on the flux-transport type of solar dynamo is briefly discussed. We consider a positive α-effect located in the overshoot layer. Our preliminary investigation supports the possibility that the location of the dynamo action is at the interface between equatorwards and polewards motions near the base of the convection zone.

Key words. Dynamo theory - Sun's magnetic field - helioseismology

1. Introduction

Mean field dynamo models have become an important tool in understanding the solar magnetic field because helioseismology gives us detailed information about the internal rotation profile which is the essential ingredient of the dynamo. However the dynamo mechanism is also influenced by the meridional flow. In the case of the solar dynamo the the most successful models assume that the essential ingredient of the dynamo model is the meridional circulation. In fact the sunspot cycle period is most probably ruled by a meridional flow located near the base of the convection zone (Hathaway et al. 2003). On the other hand, if the eddy diffusivity is low enough with a non-zero flow, significant modifications of the standard $\alpha^2\Omega$-dynamo are expected. For $\eta_{\rm T} = 10^{11}{\rm cm}^2/{\rm s}$, as it is known from the sunspot decay, the magnetic Reynolds number ${\rm Rm} = u_{\rm m}R_\odot/\eta_{\rm T}$ reaches values of the order of 10^3 for a flow of 10 m/s. As a consequence, depending on the localization of the dynamo wave both magnetic field configurations and cycle periods are determined by the strength of the meridional circulation. This possibility has recently been a subject of intense numerical investigation (Dikpati & Charbonneau 1999; Dikpati & Gilman 2001; Küker et al. 2001; Bonanno et al. 2002), where it has been shown that solutions with high magnetic Reynolds number provide correct cycle period, butterfly diagrams, magnetic phase relations and sign of current helicity, by means of a *positive* α-effect in the north hemisphere.

In this communication we shall discuss some preliminary results concerning the dependence of periods, dynamo numbers and butterfly diagram as a function of the magnetic Reynolds number for a double cell meridional circulation and for a *positive* α-effect at the base of the convection zone.

One of the findings of our investigation shows that in the high magnetic Reynolds number regime, the period of the flow tends to a constant value which is about the solar cycle time if the eddy diffusivity is of the order of $\eta_{\rm T} \approx 10^{11}{\rm cm}^2/{\rm s}$ and a positive α-effect is located at the bottom of the convection zone at variance of what has been discussed by Bonanno at al. (2005) where the α-effect was uniformly distributed in the convection zone.

2. Basic equations

In the following the dynamo equations are given for axial symmetry and the inclusion of the induction by meridional circulation. The induction equation may be written in the form

$$\frac{\partial \boldsymbol{B}}{\partial t} = {\rm rot}(\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B}) - {\rm rot}\left(\sqrt{\eta_{\rm T}}\,({\rm rot}\,\sqrt{\eta_{\rm T}}\boldsymbol{B})\right) \quad (1)$$

which includes the diamagnetism due to non-uniform turbulence.

Axisymmetry implies that the magnetic field $\mathbf{B}$ and the mean flow field $\mathbf{U}$ in spherical coordinates are given by

$$\mathbf{B} = B_\phi(r,\theta,\phi)\hat{\mathbf{e}}_\phi + \nabla \times [A(r,\theta,t)\hat{\mathbf{e}}_\phi] \quad (2)$$
$$\mathbf{U} = \mathbf{u}(r,\theta) + r\sin\theta\,\Omega(r,\theta)\hat{\mathbf{e}}_\phi \quad (3)$$

where $B_\phi(r,\theta,\phi)$ and $[A(r,\theta,t)\hat{\mathbf{e}}_\phi]$ corresponds to the toroidal and poloidal components of the magnetic field, and the meridional circulation $\mathbf{u}(r,\theta)$ and differential rotation $\Omega(r,\theta)$ to the poloidal and toroidal parts of the full flow field $\mathbf{U}$.

The meridional circulation is derived from a linear combination of stream functions, by introducing a series expansion in Legendre polynomials $P_n^{(1)}$ so that the condition div $\rho\boldsymbol{u} = 0$ is automatically fulfilled (Rüdiger 1989). A two-cell meridional circulation is thus described by

$$u_r = \frac{5}{2\rho r^2}(35\cos^4\theta - 30\cos^2\theta + 3)\Psi$$

$$u_\theta = -\frac{(70\cos^3\theta - 30\cos\theta)\sin\theta}{4\rho r}\frac{d\Psi}{dr},$$

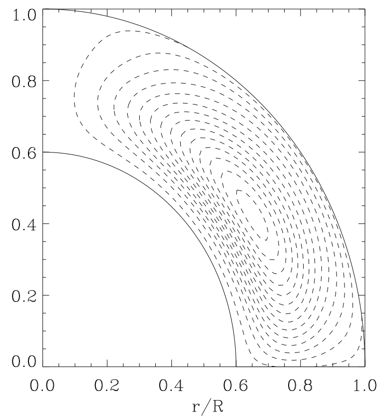

Fig. 1. Isocontour lines of the the stream function for a single cell flow of meridional circulation. Dashed lines represent a negative stream function, which implies anticlockwise meridional circulation

where a positive Ψ describes a cell circulating clockwise in the northern hemisphere, i.e. the flow is polewards at the bottom of the convection zone and equatorwards at the surface. In order to keep the flow inside the convection zone, the function Ψ must be zero at the surface and at the bottom of the convection zone. An example of stream function for a two-cell meridional flow is shown is figure.

The rotation law for the solar dynamo is obtained by the helioseismic observations and we implemented analytical model by Dikpati & Charbonneau (1999). The radial profile of $\eta_{\rm T}$ is defined as

$$\eta_{\rm T} = \eta_c + \frac{1}{2}(\eta_t - \eta_c)(1 + {\rm erf}((x - 0.7)/d))$$

where $x = r/R_\odot$ is the fractional radius, *erf* denotes the error function, η_t is the eddy diffusivity, and η_c the magnetic diffusivity beneath the convection zone. The constant d defines the thickness of the transition region to be $d = 0.025R_\odot$, and we have adopted $\eta_c/\eta_t = 0.1$.

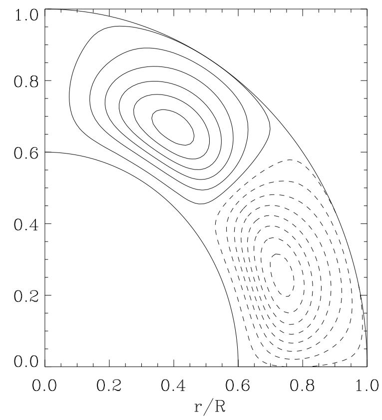

Fig. 2. Isocontour lines of the stream function for a two-cell flow of meridional circulation. Solid lines represent a positive stream function (clockwise flows) and dashed lines represent negative stream function (anticlockwise flows)

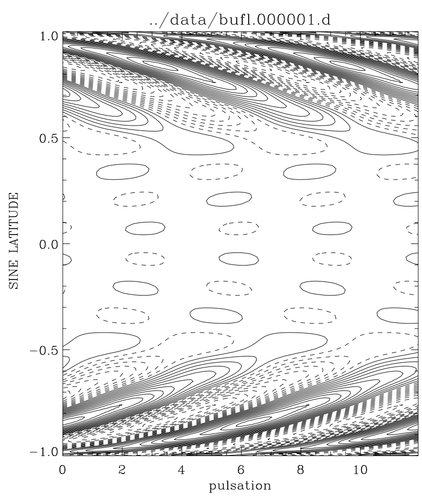

Fig. 3. Butterfly diagram for a dynamo action with no meridional circulation and differential rotation given from helioseismology. The period is 10 years and the eddy diffusivity is 2.3×10^{11} cm^2/s

The α-effect is always antisymmetric with respect to the equator, so that $\alpha = \alpha_0 \cos\theta$ and α_0 reads

$$\alpha_0 = (1 + \mathrm{erf}(\frac{x - x_\alpha}{d}))(1 - \mathrm{erf}(\frac{x - x_\beta}{d}))/4$$

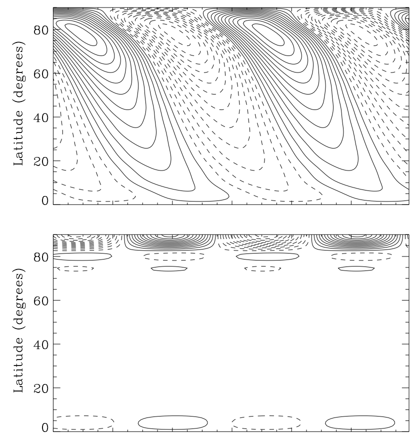

Fig. 4. Butterfly diagram for the toroidal field at 0.7 solar radii (upper panel) and for the radial field at the surface (lower panel). The strength of meridional circulation stregth is 8.3 m/s but the stream function has only one cell. The period is 26 years and the eddy diffusivity is 2.3×10^{11} cm^2/s

and x_α, x_β and d define the location and the thickness of the turbulent layer, respectively. Differently from the overshoot dynamo α_0 is *not* assumed to change its sign in the bulk of the convection zone or in the overshoot layer. In particular, we shall use a model for which a positive α-effect is located in the overshood layer, due to the magnetic instability of the tacocline. For strong magnetic fields in fact, there might be a magnetohydrodynamic instability which leads to an positive α-effect (Dikpati & Gilman 2001)

3. Results of the computation

Eq. (1) is solved with a finite-difference scheme for the radial dependence and a polynomial expansion for the angular dependence. Vacuum boundary conditions are set at the surface and perfect conductor at the inner boudary. The (infinite) system of o.d.e. is thus truncated and then solved by means of a second order accu-

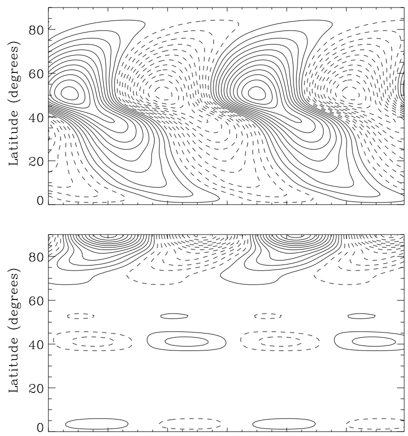

Fig. 5. Butterfly diagram for the toroidal field at 0.7 solar radii (upper panel) and for the radial field at the surface (lower panel) for the model of Fig. 4 but for a two-cell meridional circulation. The period is 16 years.

racy finite difference scheme and the basic computational task is thus to numerically compute eigenvalues and eigenvectors of a block-band diagonal real matrix of dimension $M \times n$, M being the number of mesh points and n the number of harmonics. The truncation order n can be conveniently choosen to reach the desired accuracy It is interesting to see what happens if no meridional circulation is present. The result is depicted in the butterfly diagram in Fig. 3 where no migration at low latitude is present. The presence of the flow drastically changes this picture. For a single-cell meridional circulation as in Fig. 1 the butterfly diagram is depicted in Fig. 4 where the equatorward migration is apparent, but the location of the maximum of the toroidal field is at high latitudes. If a two-cell meridional circulation is activated as in Fig. 2, the resulting butterfly diagram is shown in Fig. 5 , where it is apparent the equatorwards migration of the toroidal field starts at much lower latitudes. The periods are close to the observed one, but the presence of an additional poleward migration in the butterfly diagram in Fig. 5 above approx 45 degrees cannot be avoided if the second cell is strong enough.

4. Conclusions

In Bonanno *et al.* 2005 the α-effect was uniformly distributed in all the convection zone. In that work the periods of the resulting cycles where much longer than the observed 22 yrs value. The preliminary results of this investigation shows that it is instead possible to match the observed period with a two-cell meridional circulation if the α-effect is located in the overshoot layer. A two-cell meridional circulation provide a possible mechanism to localize the dynamo action at low latitude with a simple $\cos\theta$ angular profile for the α-effect. Further properties of a multicell meridional circulation with an overshoot α-effect will be discussed in a forthcoming paper.

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