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Parity properties of an advection dominated solar $\alpha^2\Omega$-dynamo

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Abstract. We present the results of numerical simulations of the solar dynamo both for given rotation law and meridional flow in the case of a low eddy diffusivity of the order of 10^{11} cm^2/s known from the sunspot decay. If the helioseismologically derived internal solar rotation law is considered, a model without meridional flow of high magnetic Reynolds number (corresponding to low eddy diffusivity) fails in all the three issues in comparison with the observations. However, a meridional flow with equatorial drift at the bottom of the convection zone of few meters by second can indeed enforce the equatorward migration of the toroidal magnetic field belts similar to the observed butterfly diagram but, the solution has only a dipolar parity if the (positive) α-effect is located at the base of the convection zone rather than at the top.

Key words. Dynamo theory - Helioseismology

1. Introduction

We shall here study the parity problem for solar dynamos in particular for dynamos with rather small eddy diffusivities so that the meridional flow plays an important role in advecting the toroidal magnetic field belts (Wang et al. 1991; Choudhuri et al. 1995). As the rotation law can be considered as given (known from helioseismology), we are free to vary the location of the α-effect, so that models are assumed to have a positive α-effect both at the top and at the bottom of the convection zone, as well as in the full convection zone.

The inclusion of the meridional flow $u_{\rm m}$ has a strong impact on the mean-field dynamo when the eddy diffusivity $\eta_{\rm T}$ is low. In particular, for $\eta_{\rm T}= 10^{11}{\rm cm}^2/{\rm s}$, as is known from the sunspot decay, the magnetic Reynolds number ${\rm Rm} = u_{\rm m}R_\odot/\eta_{\rm T}$ reaches values of the order of 10^3 for a flow of 10 m/s. As a consequence, depend-

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ing on the localization of the dynamo wave, a dramatic modification of both magnetic field configurations and cycle period is expected. This possibility has recently been a subject of intense numerical investigation (Dikpati & Charbonneau 1999; Küker et al. 2001), where it has been shown that solutions with high magnetic Reynolds number provide correct cycle period, butterfly diagrams, magnetic phase relations and sign of current helicity, by means of a *positive* α-effect in the north hemisphere.

In this investigation we study how the presence of the flow and the location of the turbulent layer affect the parity mode selection and the cycle period. In this respect we show that, for high magnetic Reynolds numbers of the flow, quadrupolar field configurations are more easily excited than the dipolar ones if there is no α-effect below $r/R_\odot \approx 0.8$.

2. Basic equations

In the following the dynamo equations are given with the inclusion of the induction by meridional circulation. Axisymmetry implies that the mean flow in spherical coordinates is given by

$$\boldsymbol{u} = (u_r, u_\theta, r \sin\theta \ \Omega)\,. \tag{1}$$

In our formalism the magnetic field reads

$$\boldsymbol{B} = \left(\frac{1}{r \sin\theta} \frac{\partial A \sin\theta}{\partial \theta}, -\frac{1}{r} \frac{\partial A r}{\partial r}\ ,\ B \right), \tag{2}$$

where A is the poloidal-field potential and B is the toroidal field. The dynamo equation may be written in the form

$$\begin{aligned} \frac{\partial \boldsymbol{B}}{\partial t} &= \mathrm{rot}(\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B}) \\ &\quad - \mathrm{rot}\,(\sqrt{\eta_\mathrm{T}}\,(\mathrm{rot}\,\sqrt{\eta_\mathrm{T}}\boldsymbol{B}))\,, \end{aligned} \tag{3}$$

which includes the diamagnetism due to non-uniform turbulence. If there are strong gradients of turbulence intensity, this term will dominate the transport of the mean magnetic fields.

As usual, the meridional circulation is derived from a stream function, so that the condition div $\rho\boldsymbol{u} = 0$ is automatically fulfilled. Then a series expansion in Legendre polynomials is introduced, as described in Rüdiger (1989):

$$\hat{u}_r \ = \ \frac{1}{r^2 \hat{\rho} \sin\theta}\ \frac{\partial \psi}{\partial \theta} \tag{4}$$

$$\hat{u}_\theta \ = \ -\frac{1}{r \hat{\rho} \sin\theta}\ \frac{\partial \psi}{\partial r} \tag{5}$$

A one-cell meridional circulation is described by

$$u_r = \frac{3\cos^2\theta - 1}{\hat{\rho} x^2} \psi \tag{6}$$

$$u_\theta = -\frac{\cos\theta \sin\theta}{\hat{\rho} x} \frac{d\psi}{dx}, \tag{7}$$

where ψ is the usual stream function. A positive ψ describes a cell circulating clockwise in the northern hemisphere, i.e. the flow is polewards at the bottom of the convection zone and equatorwards at the surface. For a negative ψ the flow is, as is observed, polewards at the surface. In order to keep the flow inside the convection zone, the function ψ must be zero at the surface and at the bottom of the convection zone. In order to define the strength of the flow, we shall use the values u_m of the meridional circulation at the bottom of the convection zone.

The rotation law for the solar dynamo can be considered as given by the helioseismic observations, in particular we have used the analytical model of Dikpati & Charbonneau (1999) which is characterized by the existence of a steep subrotation profile in the polar region with a thickness of about 0.05 solar radii. We then define the profile of η_T as

$$\eta_\mathrm{T} = \eta_c + \frac{1}{2}(\eta_t - \eta_c)(1 + \mathrm{erf}(40(x - 0.7))), \tag{8}$$

where $x = r/R_\odot$ is the fractional radius, *erf* denotes the error function, η_t is the eddy diffusivity, and η_c the magnetic diffusivity beneath the convection zone. The factor 40 defines the thickness of the transition region to be $0.05 R_\odot$, and we have adopted both $\eta_t/\eta_c = 0.1$ and $\eta_t/\eta_c = 0.02$.

The α-effect is always antisymmetric with respect to the equator, so that we write

$$\alpha = \alpha_0 \cos\theta \Big(1 + \mathrm{erf}(\frac{x - x_\alpha}{d})\Big)$$
$$\Big(1 - \mathrm{erf}(\frac{x - x_\beta}{d})\Big)/4, \qquad (9)$$

where $\alpha_0 > 0$ is the amplitude of the α-effect and x_α, x_β and d define the location and the thickness of the turbulent layer, respectively. Differently from the overshoot dynamo α_0 is *not* assumed to change its sign in the bulk of the convection zone or in the overshoot layer.

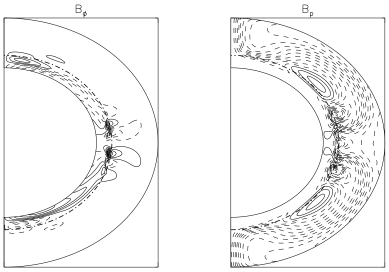

Fig. 1. Thin Alpha-effect at the bottom: Toroidal (left) and poloidal (right) antisymmetric (dipolar) field configuration for α= 4.36 cm/s, $u_{\rm m}$ = 3 m/s at t = 0 and x_α = 0.7, x_β = 0.75.

3. Numerical results

Our results can be summerized in Table I where critical α-values (cm/s) and periods (yrs) for a very thin α-layer located between $x_\alpha = 0.7$ and $x_\beta = 0.75$ for various values of the flow (m/s), for both antisymmetric and symmetric field configurations are shown. In the last line the same α-layer is located between $x_\alpha = 0.95$ and $x_\beta = 1$ and the solution is clearly of the symmetric type. The effect of the flow is dramatic in determining both the parity and the field topology of the solution. The butterfly diagram is also correct as shown in Fig. 1. where the alpha-effect was at the bottom for a very thin layer, with $u_{\rm m}$ = 12 m/s at $t = 0$ and $x_\alpha = 0.7$, $x_\beta = 0.75$

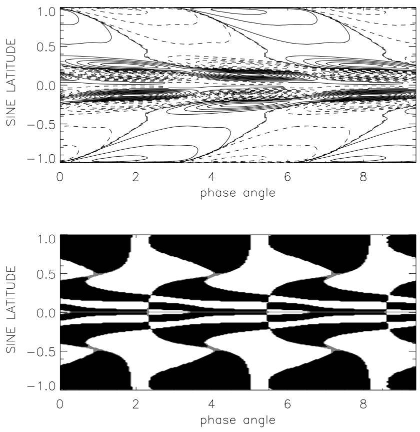

Fig. 2. Butterfly diagram and $B_r B_\phi$ sign (negative < 0) for the model presented in Fig. 1. The cycle period is 74 years.

4. Conclusions

An eddy diffusivity of 10^{11} cm^2/s provides us with a value consistent with the sunspot decay. With such a small value the magnetic Reynolds number for a meridional flow of (say) 10 m/s reaches values of order of 10^3, so that the dynamo can really be called advection-dominated. The meridional flow advects the field equatorwards producing a butterfly diagram of the observed type, which would not occur with i) a positive α-effect (in the northern emisphere), ii) the standard rotation law known from helioseismology and iii) no meridional flow (Choudhuri et al. 1995; Dikpati & Charbonneau 1999; Küker et al. 2001). As Dikpati & Gilman (2001) have stressed, these models may encounter a problem with the parity of the solution since for most of the models the quadrupolar solution is the stable one. We have used this striking fact to discuss the effect of the radial distribution of the α-effect on the parity model selection. Our conclusion is that a thin α-layer located below $r/R_\odot \approx 0.8$ selects models with correct parity prop-

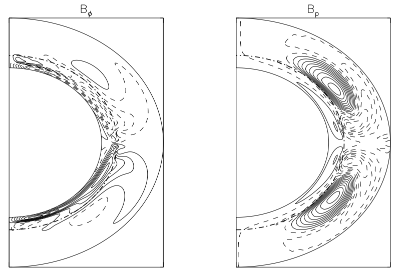

Fig. 3. Alpha-effect at the bottom for a very thin layer: toroidal (left) and poloidal (right) antisymmetric (dipolar) field configuration for α= 16.61 cm/s, $u_{\rm m}$ = 12 m/s at $t = 0$ and $x_\alpha = 0.7$, $x_\beta = 0.75$.

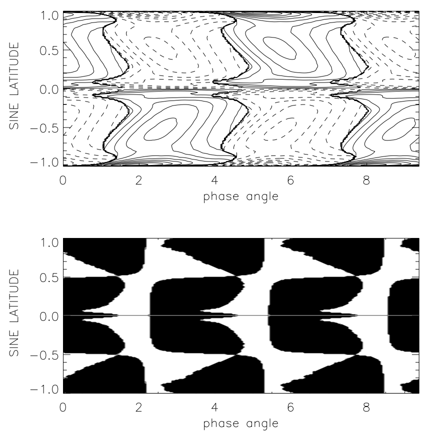

Fig. 4. Butterfly diagram and $B_r B_\phi$ sign (negative < 0) for the model presented in Fig. 3. The cycle period is 23 years.

erty, butterfly diagram phase relation and cycle periods close to the observed one.

Table 1. Critical α-values (cm/s) and periods (yrs) for a very thin α-layer located between $x_\alpha = 0.7$ and $x_\beta = 0.75$ for various values of the flow (m/s), for both antisymmetric and symmetric field configurations.

$u_{\rm m}$	$\alpha_{\rm A}$	$P_{\rm A}$	$\alpha_{\rm S}$	$P_{\rm S}$
1	**0.43**	∞	1.14	90.8
2	**1.37**	∞	1.83	∞
3	**4.36**	74	4.94	64
5	**7.53**	38	8.46	35
7	**9.22**	34	9.65	33
12	**16.61**	23	16.77	23
2	87	∞	50	283
20	43	94	40	68

This confirms the findings of Dikpati & Gilman (2001), providing further evidence for a tachocline α-effect as a promising candidate for understanding the dynamo mechanism operating in the Sun.

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