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Optical depth transition in advective accretion disks around black holes

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Abstract. We have constructed numerically global solutions of advective accretion disks around black holes that describe a continuous transition between the effectively optically thick outer and optically thin inner disk regions. We have concentrated on models of accretion flows with large mass accretion rates and we have employed a bridging formula for radiative losses at high and low effective optical depths. Contrary to the models neglecting advection, we have found that global solutions exist for the extended range of accretion rates. The presence of the effectively optically thin regions in the innermost part of accretion disks with the intermediate accretion rates (~10–50 Eddington units) results in a significant increase of the plasma temperature in those regions and this increase can be discriminated in observations.

Key words. Stars: Accretion, accretion disks — black hole physics — hydrodynamics

1. Introduction

The widely used 'standard' accretion disk model developed by Shakura (1972) and Shakura & Sunyaev (1973) is based on a number of simplifying assumptions. In particular, it is assumed that an accretion flow has a small radial velocity, small geometrical thickness, and rotates with a nearly Keplerian angular velocity. These assumptions allow to neglect by the radial gradient terms in the vertically averaged differential equations described the problem, reducing these equations to a set of algebraic equations. For the low accretion rates, $\dot{M} \ll \dot{M}_{\rm Edd}$, where $\dot{M}_{\rm Edd} = L_{\rm Edd}/c^2$ is the Eddington accretion rate, these assumptions are generally considered to be reasonable. Since the end of the seventies, however, it was realized that the inward advection of the heat neglected in the standard model becomes important in the higher accretion rate flows at $\dot{M} \gtrsim \dot{M}_{\rm Edd}$. The advection can crucially modify the properties of the innermost parts of accretion disks around black holes – the disks become hotter and thicker, and their rotation laws deviate from the Keplerian law.

Initial attempts to solve the more general disk problem included only the effects of heat advection and radial pressure gradient in mod-

Send offprint requests to: Yu. Artemova els with small values of the viscosity parameter, $\alpha = 10^{-3}$ (Paczyński & Bisnovatyi-Kogan 1981). Liang & Thomson (1980) emphasized the importance of the transonic nature of accretion flows and the influence of viscosity on the transonic accretion disk solutions was noted by Muchotrzeb (1983). Extensive investigation of accretion disk models with advection for a wide range of the parameters $\dot{M}$ and α was conducted by Abramowicz *et al.* (1988), with special emphasis on low α. Numerical solutions of accretion disks with advection have been obtained by Chen and Taam (1993) for the optically thick case and $\alpha = 0.1$, and for the optically thin case by Narayan (1996). A simplified account of advection has been attempted, either treating it like an additional algebraic term assuming a constant radial gradient of the entropy (Abramowicz *et al.* 1995; Chen *et al.* 1995; Chen 1995), or using the condition of self-similarity (Narayan & Yi 1994).

Over the last years it has become clear, that neglecting the advective heat transport at high $\dot{M}$ leads to qualitatively wrong conclusions about the topology of the family of solutions of the disk structure equations.

More specifically, the disk structure equations without the advection terms give rise two branches of solutions: optically thick and optically thin, which do not intersect if $\dot{M} < \dot{M}_{cr} \approx (0.6 - 0.9)\dot{M}_{\rm Edd}$ for $\alpha = 1$ and $M_{BH} = 10^8 M_\odot$ (Artemova et *al.* 1996). For larger accretion rates there are no solutions of these equations extending continuously from large to small radii and having Keplerian boundary conditions at the outer boundary of the disk (see also Liang & Wandel 1991; Wandel & Liang 1991; Luo & Liang 1994). It was argued by Artemova *et al.* (1996) that for accretion rates larger than $\dot{M}_{cr}$ advection becomes critically important and would allow solutions, extending all the way toward the inner disk edge, to exist also for $\dot{M} > \dot{M}_{cr}$.

Artemova *et al.* (2001) constructed transonic solutions for optically thick advective accretion disks and studied properties of singular points obtained in these solutions for a wide range of the parameters $\dot{M}$ and α. However, for some choices of $\dot{M}$ and α these solutions were not consistent in that respect that the corresponding optical depths in the inner region of the accretion disks become less than unity, which leads to violation of the optically thick disk approximation used by Artemova *et al.* (2001). The consistent treatment that describes correctly the intermediate region between the optically thick and optically thin zones should include the transition formulas for the radiative pressure and radiative flux in the equation motion and energy equation respectively (see Artemova *et al.* 1996). Most recently, Chen & Wang (2004) constructed models of advective accretion disks with $\dot{M} \lesssim \dot{M}_{\rm Edd}$ employing a bridging formula of Wandel & Liang (1991) for the optical depth transition.

The goal of the present study is to construct global numerical models of accretion disks at high accretion rates, $\dot{M} \gtrsim \dot{M}_{\rm Edd}$, taking consistently into account both the optical depth transition and advective heat transport. We study transonic models, in which we include the radial pressure gradient, radial drift velocity, and allow a non-Keplerian rotation. We employ the geometrically thin disk approximation in which the relative thickness of the disk in the considered models is always less than unity. We show that solutions extended from large radii to the inner edge of the disk can be constructed even for accretion rates considerably larger than $\dot{M}_{cr}$ and the smooth transition between optically thick and optically thin regions appears when the mass accretion rate is close to $\dot{M}_{cr}$. We demonstrate that the temperature in the optically thin region can be increased by up to a factor of 30 in comparison with the solutions obtained using the optically thick approximation. This high temperature inner disk region can explain the emission of the hard X-ray radiation observed in some X-ray sources contained accreting black holes.

2. The Model and the Method of Solution

We will consider the full set of solutions to the disk structure equations with advection icluding the optically thick, optically thin and the intermediate zones in accretion disks.

We use from now on geometric units with $G = 1$, $c = 1$, use r as the radial coordinate scaled to $r_g = M$, and scale all velocities to c. We work with the pseudo-Newtonian potential proposed by Paczyński and Wiita (1980), $\Phi = -M/(r-2)$, that provides an accurate, yet simple approximation to the Schwarzschild geometry. We normalize the accretion rate as $\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$, where $\dot{M}_{\rm Edd} = L_{\rm Edd} = 4\pi M m_p/\sigma_T$, in our units.

We use the same set of equations, ingredients and boundary conditions in our models as in Artemova et al. (2001), except for changes required by the formulae for radiative flux and radiation pressure to describe correctly the intermediate zone in the accretion disk at high accretion rates. We used the same numerical method (see Artemova et al. (2001)) to solve this modified system of algebraic and differential equations.

The following equations are therefore modified:

The vertically averaged energy conservation equation:

$$Q_{adv} = Q^+ - Q^-, \qquad (1)$$

where

$$Q_{adv} = -\frac{\dot{M}}{4\pi r}\left[\frac{dE}{dr} + P\frac{d}{dr}\left(\frac{1}{\rho}\right)\right], \qquad (2)$$

$$Q^+ = -\frac{\dot{M}}{4\pi} r\Omega\frac{d\Omega}{dr}\left(1 - \frac{l_{in}}{l}\right), \qquad (3)$$

$$Q^- = \frac{2aT^4c}{3\kappa\rho h}\left(1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_*^2}\right)^{-1}, \qquad (4)$$

are the advective energy, the viscous dissipation rate and the cooling rate per unit surface, respectively, T is the midplane temperature, κ is the opacity, a is the radiation constant and τ_0 is the Thompson opical depth, $\tau_0 = \kappa\rho h$. Here we have introduced the total optical depth to absorption, $\tau_\alpha \ll \tau$,

$$\tau_\alpha = \frac{\epsilon_{ff} + \epsilon_{fb}}{aT_c^4 c}\rho h \simeq 5.2 * 10^{21}\frac{\rho^2 T^{1/2} h}{acT^4}, \qquad (5)$$

and the effective optical depth

$$\tau_* = (\tau_0\tau_\alpha)^{1/2}.$$

Where ρ is the density and h is the half-thickness of the disk.

The equation of state for the matter consisted of a gas-radiation mixture is

$$P_{\rm tot} = P_{\rm gas} + P_{\rm rad}, \qquad (6)$$

where the gas pressure is given by

$$P_{\rm gas} = \rho\mathcal{R}T,$$

where $\mathcal{R}$ is the gas constant.

The expression for the radiation pressure is

$$P{\rm rad} = \frac{aT^4}{3}\left(1 + \frac{4}{3\tau_0}\right)\left(1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_*^2}\right)^{-1}.$$

The specific energy of the mixture is

$$\rho E = \frac{3}{2}P_{\rm gas} + 3P_{\rm rad}. \qquad (7)$$

Our method allows us to construct a self-consistent solution to the system of equations from very large radii, $r >> 100$, and down to the innermost regions of the disk.

3. Discussion and Conclusions

We have obtained unique solutions for the structure of advective accretion disks around black holes at high accretion rates, $\dot{M} \gtrsim \dot{M}_{\rm Edd}$. These solutions are global, transonic, and characterized by the optical depth transition in the inner part of accretion disks – from the optically thick to optically thin regimes. We have shown that the inclusion of the advection heat transport term into the disk structure equations allows to eliminate the problem found by Artemova *et al.* (1996) when non-advective solutions cease to exist at the inner radii where the energy balance between the local heating and radiative cooling in the effectively optically thin plasma was not satisfied. On Figure 1 we reproduce solutions using the pseudo-Newtonian potential. The solutions of this type are global, i.e. they exist for the all radial range, only if the accretion rates $\dot{m} < \dot{m}_{cr}$, where the critical accretion rate $\dot{m}_{cr}$ depends on α and the black hole mass. In particular, $\dot{m}_{cr} = 36$ in the case of $\alpha = 0.5$ and $M_{BH} = 10M_\odot$. Figure 1 shows the effective optical depth for models with $\alpha = 0.5$,

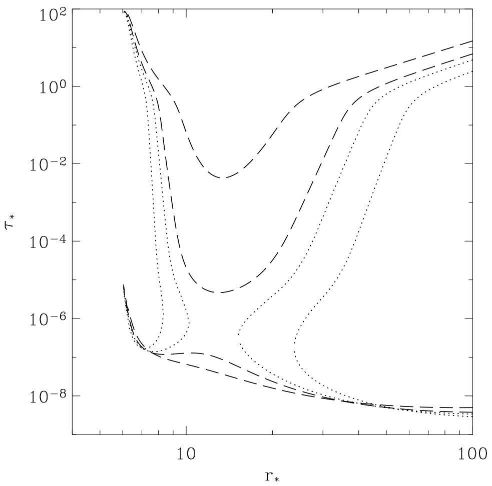

Fig. 1. Radial dependence of the effective optical depth τ_* for the models with $\alpha = 0.5$ and $M_{BH} = 10M_\odot$ constructed neglecting the radial heat advection. Dashed lines correspond to the solutions with $\dot{m} < \dot{m}_{\rm cr} = 36$. The two upper dashed lines correspond to the optically thick solutions with $\dot{m} = 20, 30$ (from top to bottom) and the two lower lines correspond to the optically thin solutions with $\dot{m} = 20, 30$ (from bottom to top). Dotted lines on the left and right panels correspond to the unphysical solutions with $\dot{m} = 36$ (left and right inner lines) and $\dot{m} = 50$ (left and right outer lines).

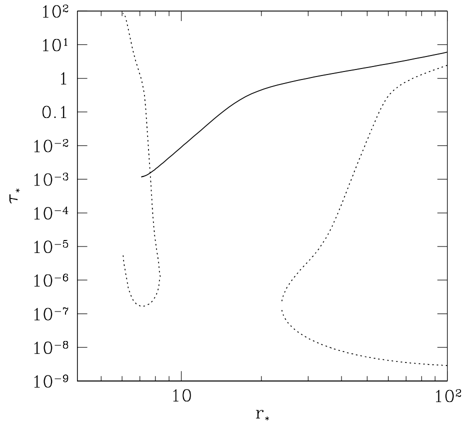

Fig. 2. Radial dependence of the effective optical depth τ_* for the model with $\alpha = 0.5$, $M_{BH} = 10M_\odot$, and $\dot{m} = 50$. The dotted lines correspond to the unphysical solutions without advection. The solid line corresponds to the global solution with advection and optical depth transition.

$M_{BH} = 10M_\odot$, and different $\dot{m}$.). The two upper dashed lines correspond to the optically thick family with $\dot{m} = 20, 30$ (counting from top to bottom), and the two lower lines correspond to the optically thin family with $\dot{m} = 20, 30$ (counting from bottom to top). Note that the effective optical depth of the optically thin family of the solutions is always very small, whereas the optical depth of the optically thick family is usually large and becomes less than unity only in the limited ranges of radii $\simeq$ 10–$40r_g$ and accretion rates $\dot{m} \simeq$ 10–30. When $\dot{m} > \dot{m}_{cr}$, there are no global solutions. Dotted lines in Figure 1 show solutions for $\dot{m} = 36$ (two inner left and right lines) and $\dot{m} = 50$ (two outer left and right lines). These solutions are clearly unphysical. Figure 2 represents the case $\dot{m} = 50 > \dot{m}_{cr}$. The dotted lines on the left and right panels show the non-advective solutions (see Figure 1) with the gap at $r \approx$ 10–$20r_g$ where the solutions do not exist, and the solid lines are the corresponding global advective solution. It was discussed above that there are no global and, therefore, physical solutions without advection for the large $\dot{m}$. The reason is that the local thermal equilibrium without the advection term, $Q^+ = Q^-$ [see eq.(1)], in the effectively optically thin plasma that radiates inefficiently can not be satisfied any more for $\dot{m} > \dot{m}_{cr}$. Therefore, in the global solutions with the optical depth transition the advection cooling term Q_{adv} is an essential and important ingredient in equation (1) to balance the viscous energy release term Q^+.

The advective models constructed using the optically thick approximation can significantly overestimate the effect of radiative cooling in the effectively optically thin regions of accretion disks. This results in the cooler interior of the disks, which then produce a softer outgoing radiation. Figure 3 shows an example of the radial dependence of the effective optical depth and Figure 4 shows the midplane temperature for the advective models with and without the optical depth transition (solid and

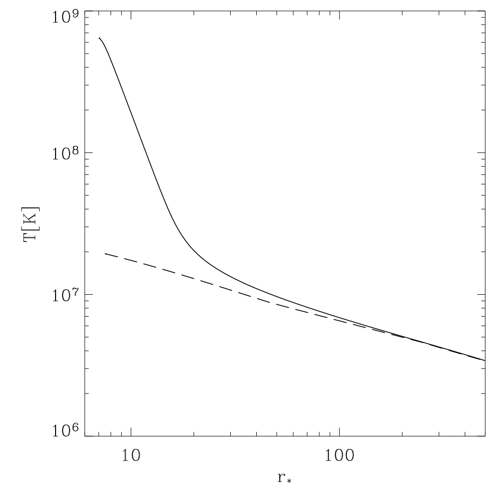

Fig. 4. Radial dependence of the midplane temperature T for the models with $M_{BH} = 10M_\odot$, $\alpha = 0.5$, and $\dot{m} = 48$. The dashed lines correspond to the advective optically thick solution and the solid lines correspond to the advective solution with the optical depth transition.

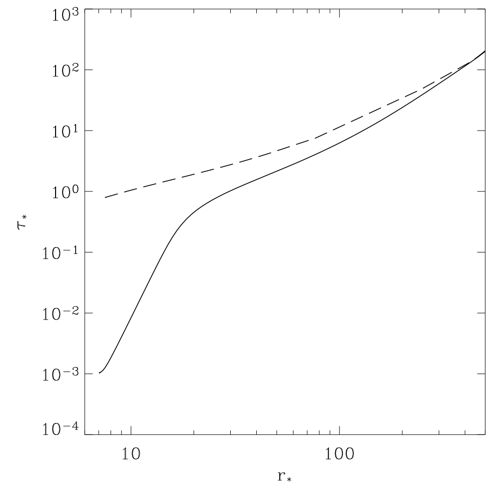

Fig. 3. Radial dependence of the effective optical depth τ_* for the model with $M_{BH} = 10M_\odot$, $\alpha = 0.5$, and $\dot{m} = 48$. The dashed lines correspond to the advective optically thick solution and the solid lines correspond to the advective solution with the optical depth transition.

dashed lines, respectively). These models are characterized by $\alpha = 0.5$, $M_{BH} = 10M_\odot$, and $\dot{m} = 48$. Note the significant increase of the temperature up to 6×10^8K and corresponding drop of the effective optical depth down to 10^{-3} inside the radius $\sim 20r_g$ in the model with optical depth transition. This is an illustrative example of the super-Eddington accretion, which can be discriminated in observations due to production of soft and hard-x-rays excesses.

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