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Testing von Zeipel's law using 2D stellar evolution

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Abstract. For spherically symmetric stars, the process of associating the observed magnitude and colour with the star's intrinsic luminosity and effective temperature is straightforward, although not without sources of error. The path is considerably less straightforward for stars with sufficient rotation that there is appreciable surface variation in both the flux and colour of the radiation emitted by the star. The traditional path for dealing with this has been von Zeipel's law, which allows one to determine the variation of the effective temperature as a function of surface location. This requires knowledge of the surface effective gravity and shape and requires that the rotation law be conservative. In this work we compare von Zeipel's law with a 2D calculation of a fairly rapidly uniformly rotating model. We find that the range of effective temperature for this model is smaller than predicted by von Zeipel's law.

1. Introduction

Rotating stars are significantly more complicated than spherically symmetric stars, and thus both our knowledge and treatment of them is not nearly as well developed. Even if we understood the interiors of the objects perfectly, we would still be faced with the problem of interpreting observations as the observed parameters depend on the unknown inclination between the observer and the rotation axis.

One of the problems introduced by the rotation of a star is in the values of the surface parameters. A spherical star can be defined by a single effective temperature and luminosity, while for a rotating star the variation of the surface flux with colatitude complicates the relationship between the observed colour, magnitude, effective temperature and luminosity. Von Zeipel's law Zei (1924) describes the relationship between the effective temperature and gravity at the surface of a star, under certain conditions. To derive this law, one assumes the centrifugal force is conservative, so that the total potential balances the pressure forces. Hydrostatic equilibrium implies that the properties of the gas are constant on equipotential surfaces (Φ = const), which in turn implies that the flux, assumed to be radiative, is emitted perpendicular to an equipotential surface. From this, it can be shown that the surface flux is proportional to the gradient of the effective potential, i.e. F $\propto \frac{d\Phi}{dn_i}$. Combining this with the definition of effective temperature, we recover von Zeipel's Law:

$$T_e^4 \propto F \propto \frac{d\Phi}{dn_i} \propto g_{eff} \tag{1}$$

This law is typically used to calculate the effective temperature distribution of one-dimensional rotating stellar models. In this paper, we have investigated the accuracy of von

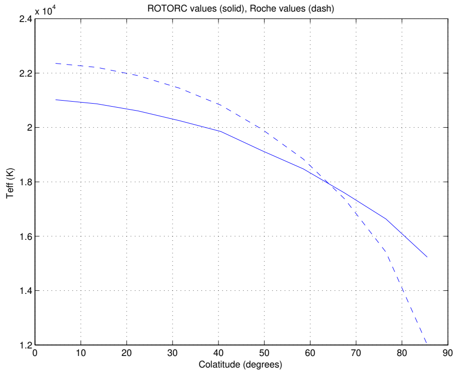

Fig. 1. Surface temperature of ZAMS model - solid = ROTORC, dashed = von Zeipel's predictions. The two models are scaled to have the same total luminosity.

Zeipel's law for a case of very rapid rotation using a 2D stellar evolution code, `ROTORC` (Deup , 1990; **?**).

2. Models

We created ZAMS models using our stellar evolution code. Because this code is 2D, it provides us directly with the effective temperature and gravity as a function of colatitude. We then compared the results of our models with the predictions of von Zeipel's law. We have performed this comparison for a 6.5 $M_\odot$ model with near critical uniform rotation (Ω/Ω_{crit} = 0.89). We examined this model on the ZAMS and at a more evolved point on the main sequence. We find that `ROTORC` predicts a smaller range of effective temperatures than von Zeipel's law (see Fig. 1). In this figure, the temperatures calculated by von Zeipel's law are normalized such that the total luminosity produced by the `ROTORC` model and von Zeipel's law is the same. We believe the difference between the two models occurs because von Zeipel's law holds that the surface is an equipotential surface with the temperature constant on this surface. However, the variation in the effective temperature along the surface is expected to correlate in some way to the variation of actual surface temperature along the surface, producing a contradiction. The departures of the temperature from uniformity on equipotential surfaces going in from the surface of the model are the same order of magnitude as the differences found by integrating model envelopes (in the stellar interiors sense) from the surface inward with the surface temeprature coupled to the effective temperature and the surface temperature treated as a free parameter.

The results for the more evolved model are similar, although the difference between the model results and the predictions of von Zeipel's law are smaller.

3. Conclusion

We find that von Zeipel's law predicts a larger range of effective temperatures over the surface of a rotating star than that calculated by our stellar structure models. These differences are largest for ZAMS models, and decrease over the course of the stars lifetime as the surface expands and becomes more spherical.

Although von Zeipel's law is a good approximation for slowly rotating stars, we find that in rapidly rotating stars, it must be used with caution, as its results may deviate from more realistic models.

References

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