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The projection factor for Cepheid distances determination

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Abstract. The distance of galactic Cepheids can be derived through the interferometric Baade-Wesselink method. In this method, the projection factor which is the ratio of the pulsation velocity (V_{puls}) to the radial velocity deduced from line profile position (V_{rad}) is of crucial importance. Using a geometric model, we compare different definitions of the radial velocity. The so-called barycentric velocity, which is independent of the Full-Width at Half-Maximum (FWHM) of the line and the rotation, is certainly the most adapted for Cepheids distance determination. However, the projection factor remains sensitive to velocity gradient effects. Then, a self-consistent and time-dependent model of the star δ Cep is computed in order to study the dynamical structure of its atmosphere together with the induced line profile. Different kinds of pulsation velocities are then derived. In particular, we deduce a suitable average value for the projection factor related to different observational techniques, such as spectroscopy, and spectral-line or wide-band interferometry. We show that the impact of velocity gradient on the average projection factor, and consequently on the final distance deduced from this method, is of the order of 6%. Finally, we point out that spectro-interferometry can provide a new way to constrain directly the projection factor by observations.

Key words. Stars: Distances – Stars: Cepheids – Stars: Atmosphere

1. Scientific rationale

The distance of galactic Cepheids can be derived through the interferometric Baade-Wesselink method (IBW). Recent observations with VINCI/VLTI have provided the distance of seven Cepheids (Kervella et al. 2004a Kervella (2004)). From these results, it is possible to calibrate in a new geometric way the Period-Luminosity (Kervella et al. 2004b Kervella 2 (2004)) and the Surface-Brightness relations (Kervella et al. 2004c Kervella 3 (2004)), which are both very important for extragalactic distances determination.

The IBW method combines spectrometric and interferometric observations. The interferometric measurements lead to limb-darkened angular diameter estimations over the whole pulsation period, while the stellar radius variations can be deduced from the integration of the pulsation velocity (V_{puls}). The latter is linked to the observational velocity (V_{rad}) deduced from line profiles through the so-called

Send offprint requests to: N. Nardetto *projection factor p*. In this method angular and linear estimations of the Cepheid radius have to correspond to the same layer in the star to provide a correct estimate of the distance.

2. The impact of the radial velocity

In this section we compare different definitions of the radial velocity and their impact on the projection factor. For this purpose we use a geometric model of a limb-darkened pulsating star in rotation with a one-layer atmosphere. Our model has 4 parameters:

- The limb-darkening of the star : we consider a linear law for the continuum-intensity profile of the star defined by

  $$I(\cos(\theta)) = 1 - u_V + u_V \cos(\theta),$$

  where u_V corresponds to the limb-darkening of the star in the visible. θ is the angle between the normal of the star and the line-of-sight.
- The projected rotation velocity, $V_{\rm rot} \sin i$, where i the angle between the line-of-sight and the rotation axis (in km.s^{-1}).
- The pulsation velocity (in km.s^{-1}) defined by

  $$V_{\rm puls}(\phi_i) = V_{\rm max} \cos(2\pi\phi_i),$$

  with a typical value for the maximum pulsation velocity of $V_{\rm max} = 30$ km.s^{-1}.
- A width for the spectral (in Å), hereafter CFWHM. It is the FWHM of the line with no pulsation and rotation velocities. It is supposed to be *constant* with the pulsation phase.

The velocity field is a combination of pulsation and rotation velocities. By Doppler effect this field can be transposed into wavelengths, and weighted by the surface brightness (limb-darkening) to obtain the weighting of the spectral line. We have then to convolve with the static profile to obtain the synthetic spectral line profile.

We consider a typical limb-darkening of $u_V = 0.7$. For different sets of CFWHM and rotation, we derive the synthetic spectral line profile. From this profile we define three radial velocities :

- The velocity associated to the minimum of the line ($RV_{\rm m}$).
- The velocity derived from the fit of a gaussian on the line profile. The central position of the gaussian provide the gaussian velocity ($RV_{\rm g}$).
- The barycentric velocity which is the first moment of the spectral line profile ($RV_{\rm b}$).

The corresponding projection factors, respectively noted, $p_{\rm m}$, $p_{\rm g}$ and $p_{\rm b}$ are plotted on Fig. 1ab. We note that the barycentric projection factor is affected neither by the rotation nor the CFWHM, as was already pointed out by Burki et al. 1982 Burki (1982). It depends only marginally on the limb-darkening. We thus emphasize that the barycentric projection factor is certainly the best one to use in the context of the Interferometric Baade-Wesselink method. More details about the barycentric projection and the line asymmetry can be found in Nardetto et al. 2005 Nardetto 2 (2005). Dynamics effects, which are not taken into consideration in our geometric model, have however an important impact on the projection factor through the pulsation velocity definition.

3. The impact of the pulsation velocity

We apply for δ Cep (HD 213306) a nonlinear self-consistent hydrodynamical code. The model needs only 4 input parameters: the luminosity (L), the effective temperature ($T_{\rm eff}$), the mass (M) and the chemical composition X and Y. The model runs until it reaches its limit cycle. Since the main stellar quantities of δ Cep are still uncertain, we tried several sets of L, $T_{\rm eff}$, M, X and Y in order to get suitable observational quantities such as the pulsation period, the average radius of the star, bolometric and radial velocity curves, and line profiles. This leads to the following set for the model: $M = 4.8\,M_{\odot}$, $L = 1995\,L_{\odot}$, $T_{\rm eff} = 5877K$, $X = 0.70$ and $Y = 0.28$. The limit cycle is reached with the pulsation period of 5.419 days. Bolometric and radial velocity amplitudes are respectively $\Delta m_{\rm bol} = 0.85$ mag and $2K = 35$ km s^{-1}. The relative radius amplitude

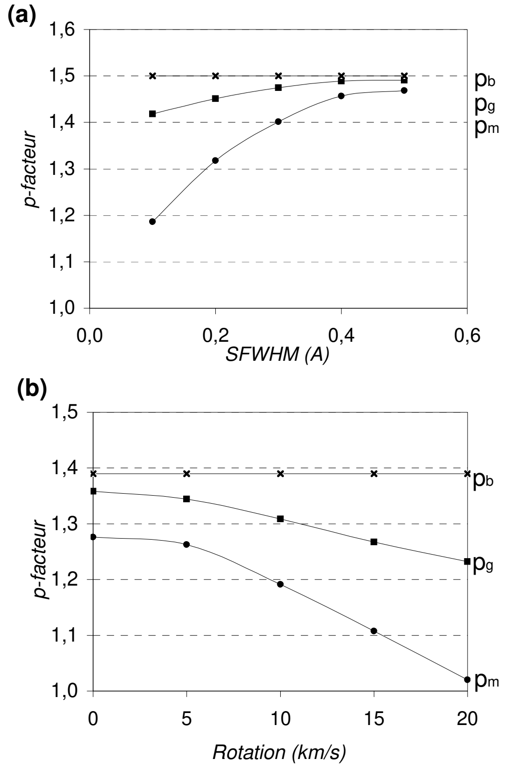

Fig. 1. Different definitions of the projection factor as a function of the CFWHM and rotation. The limb-darkening is fixed to $u_V = 0.7$. We note the low sensitivity of the barycentric projection factor which is thus the most adapted to Cepheid distance determination.

at the surface is $\Delta R/R = 10\,\%$. The mean photospheric radius is about $\overline{R} = 43.5\,R_\odot$

Radiative transfer in the line is then solved in the frame of this hydrodynamical model to provide line profiles. For the present study, which is a first step, we have arbitrarily considered the metallic line Fe I 6003.012 Å. Therefore, we can compare the velocity in a given mass zone ($v_{\rm puls}$) with the velocity measured from the synthetic line profile ($v_{\rm rad}$).

Three different pulsation velocities have to be considered, corresponding to different type of observation.

Firstly, from a spectroscopic point of view, one considers the gas velocity associated to the barycenter of the line forming region. Hence, we use the definition:

$$V_{\rm puls_{(s)}} = v(\tau_l = \frac{2}{3}) \quad (1)$$

where τ_l is the optical depth at the center of the line and "(s)" means "Spectroscopy".

Secondly, for interferometric observations in one particular line, it is better to consider the velocity of optical layers corresponding to an optical depth of $\tau_l = 2/3$. It is not the gas velocity that is considered here but the *velocity of the optical layer* deduced from the pulsation model, defined by:

$$V_{\rm puls_{(il)}} = \frac{\partial R(\tau_l = 2/3)}{\partial \phi} \quad (2)$$

where "(il)" is for "Interferometry in one Line".

Similarly, for interferometric observations in a wide band, the most appropriate pulsation velocity is the one associated to the photospheric layer that corresponds, by definition, to $\tau = 2/3$ in the continuum:

$$V_{\rm puls_{(ic)}} = \frac{\partial R(\tau_c = 2/3)}{\partial \phi} \quad (3)$$

where "(ic)" is for "Interferometry in the Continuum". Note that we consider here the continuum next to the line.

We derive a suitable average value for the projection factor related to the different observational techniques : spectrometry [$p_{\rm s} = 1.35$], and spectral-line [$p_{il} = 1.32$] or wide-band [$p_{\rm ic} = 1.27$] interferometry. These results are provided using the gaussian fit method to derive the radial velocity. We show that the impact on the average projection factor, and consequently on the final distance deduced from this method, is of the order of 6%. Note that this result is also available for other radial velocity definitions. This systematic error is comparable to the statistical error corresponding to interferometric observations : with VINCI/VLTI we obtain a relative precision on the ℓ Car distance of 5%. This theoretical result has been confirmed observationally, using HST trigonometric parallaxe measurements of δ Cep by Mérand et al. Merand (2005).

Otherwise, the influence of the time dependence of the projection factor is not significant as the error in the final distance is of the order of 0.2% (see Nardetto et al. 2004 Nardetto (2004)). However, Sabbey et al. 1995 Sabbey (1995) found an impact on the radius of 6%.

4. The projection factor and the spectro-interferometry

The 4% effect we found between the $p_{il} = 1.32$ and $p_{ic} = 1.27$ values of the projection factor corresponds also to an effect of 4% between the photospheric and the line-forming region radii. Can this 4% effect be detected through spectro-interferometry? The same model is used to derive intensity profiles in different spectral lines together with the corresponding wavelength-dependent visibility curve. Our model predicts that the spectral lines considered form gradually in different part of the stellar atmosphere (see Fig. 2ab). For a spectroscopic resolution of $R = 90000$ in visible, we find in most favourable case, at maximum contraction velocity ($\phi = 0.77$; $V_{puls} = -23$km.s^{-1}) and for a projected baseline of 65 meters, a signature on the visibility curve of about 19%, as indicated by Fig. 2c. Note that the visibility curves have been arbitrarily shifted in wavelength and visibility for relative comparison. Spectral lines formed in the upper part of the atmosphere lead to an important effect on the visibility curve while lines forming in the lower part, close to photosphere, lead to a minor effect on the visibility. There is a correspondence between the line-forming region and the amplitude of the visibility signature.

Such observations would lead to a new geometric view of the velocity gradient in the atmosphere of the star, and in particular would provide important information on spectral lines contribution functions. This is also a new way to constrain the projection factor directly by observations. More details can be found in Nardetto et al. 2005 Nardetto 3 (2005).

5. Conclusion

The projection factor is currently the most important bias of the IBW method for Cepheid distance determination. First, we find that the barycentric radial velocity is the most adapted to Cepheid distance determination as it is independent of the FWHM and the rotation of the star. Second, velocity gradient has an impact on the projection factor of about 6%. Finally, spectro-interferometric observations should provide a new interesting view of the Cepheid atmosphere and thus of the projection factor.

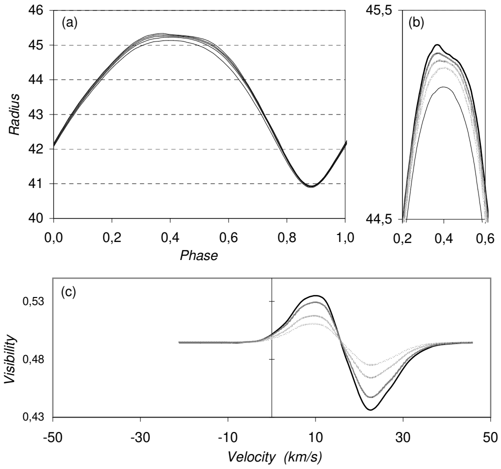

Fig. 2. (a) The photospheric radius defined as $\tau_c = 2/3$ and the line-forming region radius defined as $\tau_l = 2/3$ are represented as a function of the pulsation phase. We have (down to up) : the photospheric radius, and the radius corresponding to the lines Ni I 6378.2470 Å , Fe I 6380.7433 Å , Fe I 6056.0047 Å and Fe I 6003.0123 Å . (b) The same as (a) but zoomed around phase of maximum expansion of the star. (c) The visibility is represented as a function of the velocity in the case of the four spectral lines ($\phi = 0.77$). The resolution is $R = 90000$.

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