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The dark side of gravity

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Abstract. The discovery of the cosmic acceleration has made it clear that we have very little direct information on the global properties of our universe. The main goal of research in the next decade should be the reconstruction of the space-time metric at the background level and at first order. This will be a crucial step on the road to understanding the nature of dark energy.

1. Introduction

The observed accelerated expansion of the Universe is considered as the main mystery in modern cosmology and one of the major issues confronting theoretical physics at the beginning of the new Millennium. Although a plethora of models have been constructed, none of them is really appealing on theoretical grounds. An alternative approach in this situation is to construct general parametrizations of the dark energy, in the hope that measuring these parameters will give us some insight into the mechanism underlying the dark energy phenomenon.

A successful example for such a phenomenological parametrization in the dark energy context is the equation of state parameter of the dark energy component, $w \equiv p/\rho$. If we can consider the Universe as evolving like a homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) universe, and if the dark energy is not coupled to anything except through gravity, then $w(z)$ completely specifies its evolution. The dark energy (or anything else) is described by the homogeneous energy density $\rho_{\rm DE}$ and the isotropic pressure $p_{\rm DE}$, corresponding to the T^0_0 and T^i_i elements respectively in the energy momentum tensor in the rest-frame of the dark energy. Any other non-zero components would require us to go beyond the FLRW description of the Universe. The evolution of ρ is then governed by the "covariant conservation" equation $T^{\nu}_{\mu;\nu} = 0$ which is just

$$\dot{\rho}_{\rm DE} = -3H(\rho_{\rm DE} + p_{\rm DE}) = -3H(1+w)\rho_{\rm DE}. \quad (1)$$

As long as $H \neq 0$ we can choose the evolution of $\rho_{\rm DE}$ through the choice of $p_{\rm DE}$ or equivalently the choice of w. It is usually more convenient to quote w since it is independent of the absolute scale of ρ and p and many fluids give rise to simple expressions for w, for example $w = 0$ for pressureless dust, $w = 1/3$ for a radiation fluid, or $w = -1$ for a cosmological constant.

At the background level, the only observationally accessible quantity is the expansion rate of the universe H, given by the Friedmann equation,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_{\rm DE})\,. \qquad (2)$$

For simplicity we neglect radiation and assume that space is flat. The distances are integrals of $1/H$, and H can be directly measured with some methods like baryonic acoustic oscillations (BAO) (Eisenstein et al. 2005) or the dipole of the supernova distribution (Bonvin et al. 2006). The relative abundance of matter today Ω_m could also be measured, but at the moment this is not possible directly, as we have not yet been able to detect the dark matter in experiments. The estimate of Ω_m through dynamical means (virial theorem or lensing) relies on assuming Newtonian gravity. Thus, at this level only H is directly measurable, and we would like to infer w, Ω_m and $\Omega_{\rm DE} = 1 - \Omega_m$.

2. The dark sector

Assuming that we have a perfect measurement of H, we can then directly derive an expression for w:

$$w(z) = \frac{H(z)^2 - \frac{2}{3}H(z)H'(z)(1+z)}{H_0^2\Omega_m(1+z)^3 - H(z)^2} \qquad (3)$$

This expression exposes an awkward problem with our data: since we do not know Ω_m, we find a solution for w for *every* choice of Ω_m. We cannot therefore measure both w and Ω_m simultaneously without approximations. Consequently, we are left with a one-parameter family of possible w's (Kunz 2007).

This may surprise the reader, since she or he may have thought that $\Omega_m \approx 0.25$ from e.g. supernovae Ia observations. However, this conclusion requires assumptions on the nature of the dark energy, and so let us spend a few lines looking at them. One possible assumption is to impose $w = -1$, i.e. to demand that the dark energy is a cosmological constant. Alternatively, we would reach similar limits on Ω_m by being slightly more general and allowing w to be a free constant. Both of these are very strong assumptions if we wanted to actually learn something about the dark energy from the data! Indeed, we should conclude that any dark energy analysis that uses only data based on measurements of "background" quantities derivable from H like distances or ages *must* find no constraint on Ω_m, or else the parametrization of w is not sufficiently general[1].

Often more data is included in the analysis, for example the angular power spectrum of temperatures anisotropies in the cosmic microwave background (CMB). This data not only constrains the expansion history of the Universe, but also the clustering of the fluids. Under the additional (and also strong) assumption that the dark energy does not cluster significantly, we are then able to separate the dark matter and the dark energy in this case (Kunz 2007).

To include the CMB data we need to improve our description of the universe, by using perturbation theory. If we work in the Newtonian gauge, and we neglect the perturbations introduced by gravitational waves and vorticity, we must add two gravitational potentials ϕ and ψ to the metric. They can be considered as being similar to H in that they determine completely at first order the description of the space-time (but they are functions of scale as well as time). Also the energy momentum tensor becomes more general, and ρ is complemented by perturbations $\delta\rho$ as well as a velocity V_i. The pressure p now can also have perturbations δp and there can further be an anisotropic stress π.

The reason why we grouped the new parameters in this way is to emphasize their role: at the background level, the evolution of the universe is described by H, which is linked to ρ by the Einstein equations, and p controls the evolution of ρ but is a priori a free quantity describing the physical properties of the fluid. Now in addition there are ϕ and ψ describing the Universe, and they are linked to $\delta\rho_i$ and V_i of the fluids through the Einstein equations. δp_i and π_i in turn describe the fluids. Actually, there is a simplification: the total

[1] Adding even more freedom to the dark energy, like allowing for couplings to the dark matter, makes the degeneracy even worse.

anisotropic stress π directly controls the difference between the potentials, $\phi - \psi$.

3. Dark energy phenomenology

We have seen in the previous section that a general dark energy component can be described by phenomenological parameters even at the level of first order perturbation theory ("parameter" is used here in a general way: a function can be represented as a very large number of parameters). This description adds two new parameters δp and π, which are both functions of scale as well as time. These parameters fully describe the dark energy fluid, and they can in principle be measured.

However, recently much interest has arisen in modifying GR itself to explain the accelerated expansion without a dark energy fluid (Amendola et al. 2007). What happens if we try to reconstruct our parameters in this case? Is it possible at all?

Let us assume that the (dark) matter is three-dimensional and conserved, and that it does not have any direct interactions beyond gravity (after all, dark matter is in principle separately observable through direct detection). We assume further that it and the photons move on geodesics of the same (possibly effective) 3+1 dimensional space-time metric. In this case we can write the modified Einstein equations as

$$X_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{4}$$

where the matter energy momentum tensor still obeys $T^{\nu}_{\mu;\nu} = 0$. While in GR this is a consequence of the Bianchi identies, this is now no longer the case and so this is an additional condition on the behavior of the matter.

In this case, we can construct $Y_{\mu\nu} = X_{\mu\nu} - G_{\mu\nu}$, so that $G_{\mu\nu}$ is the Einstein tensor of the 3+1 dimensional space-time metric and we have that

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} - Y_{\mu\nu}. \tag{5}$$

Up to the prefactor we can consider Y to be the energy momentum tensor of a dark energy component. This component is also covariantly conserved since T is and since G obeys the Bianchi identities. The equations governing the matter are going to be exactly the same, by construction, so that the effective dark energy described by Y mimics the modified gravity model.

By looking at Y we can then for example extract an effective anisotropic stress and an effective pressure perturbation and build a dark energy model which mimics the modified gravity model and leads to *exactly* the same observational properties (Kunz & Sapone 2007).

This is both good and bad. It is bad since cosmology cannot directly distinguish dark energy from modified gravity [2]. However, it is good since there is a clear target for future experiments: their job is to measure the two additional functions describing Y as precisely as possible.

4. Forecasts for future experiments

As an example, we show in Fig. 1 two graphs from (Amendola et al. 2008). Here a different choice was made for the parametrization of the extra dark energy freedom: the logarithmic derivative of the dark matter perturbations were characterized by $\Omega_m(z)^\gamma$, and the deviations of the lensing potential $\phi + \psi$ from a fiducial cosmology with unclustered dark energy (chosen arbitrarily as a reference point) with a function Σ. Because of space constraints, we refer the reader to (Amendola et al. 2008) for more details.

5. Conclusions

We have shown in these proceedings that both dark energy and modified gravity cosmologies can be described at the level of first-order perturbation theory by adding two functions to the equation of state parameter $w(z)$. This allows to construct a phenomenological parametrization for the analysis of e.g. CMB data, weak lensing surveys or galaxy surveys, which depend in essential ways on the behavior of the perturbations.

[2] Although there could be clear hints, e.g. a large anisotropic stress would favor modified gravity since in these models it occurs generically while scalar fields have $\pi_i = 0$.

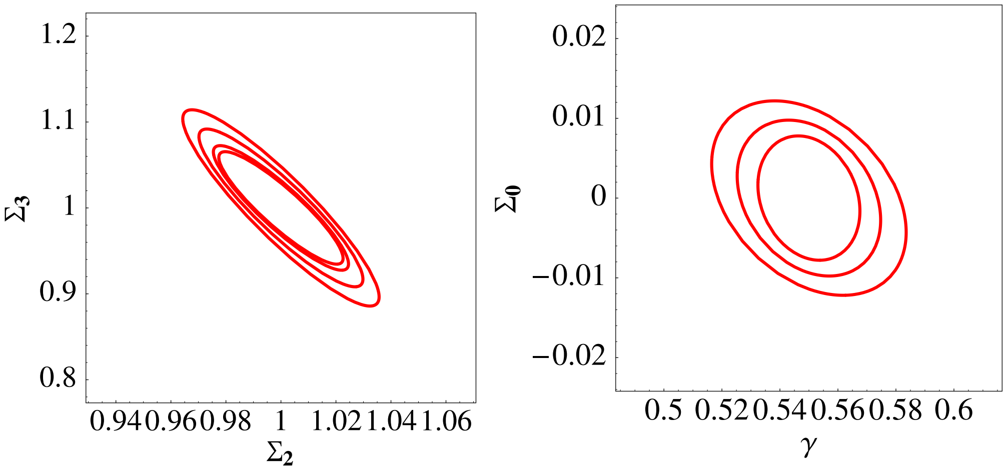

Fig. 1. Forecast of how a future weak lensing survey could constrain the behavior of the dark energy or modified gravity at the perturbation level.

Different choices are possible for these two functions. For example, one can directly use the gravitational potentials $\phi(k,t)$ and $\psi(k,t)$ and so describe the metric[3]. Alternatively one can use the parameters which describe the dark energy, the pressure perturbation $\delta p(k,t)$ and the anisotropic stress $\pi(k,t)$. The pressure perturbation can be replaced by a sound speed c_s^2, which then has to be allowed to depend on scale and time. As argued above, these parameters can *also* describe modified gravity models, in which case they do of course not have a physical reality. Also other choices are possible, the important message is that *only two new functions* are required (although they are functions of scale k and time t). Together with $w(z)$, they span the complete model space for both modified gravity and dark energy models in the cosmological context (i.e. without direct couplings, and for 3+1 dimensional matter and radiation moving on geodesics of a single metric). By measuring them, we extract the *full information* from cosmological data sets to first order. These phenomenological functions are useful in different contexts. Firstly they can be used to analyze data sets and to look for *general* departures from e.g. a scalar field dark energy model. If measured, they can also give clues to the physical nature of whatever makes the expansion of the Universe accelerate. Finally, they are useful to forecast the performance of future experiments in e.g. allowing to rule out scalar field dark energy, since for this explicit alternatives are needed.

[3] This would correspond to use $H(z)$ rather than $w(z)$ to describe the dark energy at the background level.

References

Amendola, L., Gannouji, R., Polarski, D., & Tsujikawa, S., 2007, Phys. Rev. D, 75, 083504

Amendola, L., Kunz, M., & Sapone, D., 2008, JCAP, 04, 013

Bonvin, C., Durrer, R., & Kunz, M., 2006, Phys. Rev. Lett., 96, 191302

Eisenstein, D., et al., 2005, ApJ, 633, 560

Kunz, M., 2007, astro-ph/0702615

Kunz, M., & Sapone, D., 2007, Phys. Rev. Lett., 98 121301