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Fundamental constants, general relativity and cosmology

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Abstract. The tests of the constancy of the fundamental constants are tests of the local position invariance and thus of the equivalence principle. We summarize the links of the studies on fundamental constants to the tests of general relaitivity and their cosmological importance.

Key words. Fundamental constants, general relativity, cosmology: theory

1. Introduction

1.1. Generalities

Physical theories usually introduce constants, i.e. numbers that are not, and by construction can not be, determined by the theory in which they appear. They are contingent and can only be experimentally determined and measured. It is an important property of these constants that they can actually be measured.

These numbers have to be assumed constant for two reasons. First, from a theoretical point of view, we have no evolution equation for them (since otherwise they would be fields) and they cannot be expressed in terms of other more fundamental quantities. Second, from an experimental point of view, in the regimes in which the theories in which they appear have been validated, they should be constant at the accuracy of the experiments, to ensure the reproducibility of these experiments. This means that testing for the constancy of these parameters is a test of the theories in which they appear and allow to extend the knowledge of their domain of validity.

Indeed, when introducing new, more unified or more fundamental, theories the number of constants may change so that the list of what we call fundamental constants is a time-dependent concept and reflects both our knowledge and ignorance (Weinberg, 1983). Today, gravitation is described by general relativity, and the three other interactions and whole fundamental fields are described by the standard model of particle physics. In such a framework, one has 22 unknown constants [the Newton constant, 6 Yukawa couplings for the quarks and 3 for the leptons, the mass and vacuum expectation value of the Higgs field, 4 parameters for the Cabibbo-Kobayashi-Maskawa matrix, 3 coupling constants, a UV cut-off to which one must add the speed of light and the Planck constant; see e.g. Hogan (2000)].

Since any physical measurement reduces to the comparison of two physical systems, one of them often used to realize a system of units, it only gives access to dimensionless numbers. This implies that only the variation

Send offprint requests to: Jean-Philippe Uzan of dimensionless combinations of the fundamental constants can be measured and would actually also correspond to a modification of the physical laws [see e.g. Uzan (2003), Ellis and Uzan (2005)]. Changing the value of some constants while letting all dimensionless numbers unchanged would correspond to a change of units. It follows that from the 22 constants of our reference model, we can pick 3 of them to define a system of units (such as e.g. c, G and h to define the Planck units) so that we are left with 19 unexplained dimensionless parameters, characterizing the mass hierarchy, the relative magnitude of the various interactions etc.

Indeed, this number can change with time and with our knowledge of physics. For instance, we know today that neutrinos have to be somewhat massive. This implies that the standard model of particle physics has to be extended and that it will involve at least 7 more parameters (3 Yukawa couplings and 4 CKM parameters). On the other hand, this number can decrease, e.g. if the non-gravitational interactions are unified. In such a case, the coupling constants may be related to a unique coupling constant α_U and a mass scale of unification M_U through $\alpha_i^{-1}(E) = \alpha_U^{-1} + (b_i/2\pi)\ln(M_U/E)$, where the b_i are numbers which depends on the explicit model of unification. This would also imply that the variations, if any, of various constants will be correlated.

1.2. Constants and general relativity

The tests of the constancy of fundamental constants take all their importance in the realm of the tests of the equivalence principle (Will, 1993). This principle, which states the universality of free fall, the local position invariance and the local Lorentz invariance, is at the basis of all metric theories of gravity and implies that all matter fields are universally coupled to a unique metric $g_{\mu\nu}$ which we shall call the physical metric, $S_{\rm matter}(\psi, g_{\mu\nu})$. The dynamics of the gravitational sector is dictated by the Einstein-Hilbert action $S_{\rm grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}R_*\mathrm{d}^4x$. General relativity assumes that both metrics coincide, $g_{\mu\nu} = g^*_{\mu\nu}$.

The test of the constancy of constants is a test of the local position invariance hypothesis and thus of the equivalence principle. Let us also emphasize that it is deeply related to the universality of free fall (Dicke, 1964) since if any constant c_i is a space-time dependent quantity so will the mass of any test particle. It follows that the action for a point particle of mass m_A is given by

$$S_{p.p.} = -\int m_A[c_j]c\sqrt{-g_{\mu\nu}(x)v^\mu v^\nu}\mathrm{d}t$$

with $v^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}t$ so that its equation of motion is

$$u^\nu\nabla_\nu u^\mu = -\left(\frac{\partial\ln m_A}{\partial c_i}\nabla_\beta c_i\right)(g^{\beta\mu} + u^\beta u^\mu). \qquad (1)$$

It follows that a test body does not enjoy a geodesic motion and experience an anomalous acceleration which depends on the sensitivity $f_{A,i} \equiv \partial\ln m_A/\partial c_i$ of the mass m_A to a variation of the fundamental constants c_i. In the Newtonian limit, $g_{00} = -1 + 2\Phi_N/c^2$ so that $\mathbf{a} = \mathbf{g}_N + \delta\mathbf{a}_A$ with the anomalous acceleration $\delta\mathbf{a}_A = -c^2\sum_i f_{A,i}\left(\nabla c_i + \frac{\mathbf{v}_A}{c^2}\dot{\alpha}_i\right)$. Such deviations are strongly constrained in the Solar system and also allow to bound the variation of the constants (Dent, 2008).

1.3. Cosmology

This allows to extend tests of the equivalence, and thus tests of general relativity, on astrophysical scales. Such tests are central in cosmology in which the existence of a dark sector (dark energy and dark matter) is required to explain the observations. Universality classes of dark energy models have been defined (Uzan *et al.*, 2004, Uzan, 2007) and the constants allow to test some of these classes, hence complementing other tests of general relativity on astrophysical scales (Uzan, 2007, 2009, 2010, Uzan and Bernardeau, 2001) and of the hypothesis of the cosmological model (Uzan *et al.*2008).

Necessity of theoretical physics in our understanding of fundamental constants and on deriving bounds on their variation is, at least, threefold. (i) It is necessary to understand and to model the physical systems used to set the constraints (and to determine the effective parameters that can be observationally constrained to a set of fundamental constants); (ii) it is necessary to relate and compare different constraints that are obtained at different space-time positions (this requires a space-time dynamics and thus to specify a model); (iii) it is necessary to relate the variation of different fundamental constants through e.g. unification.

This text summarizes these three aspects by first focusing, in § 2, on the various physical systems that have been used, in § 3, on the theories describing varying constants while § 4 summarizes the links with cosmology.

2. Physical systems and constraints

2.1. Physical systems

The various physical systems that have been considered can be classified in many ways [see Uzan (2003,2004,2009) for reviews and Uzan and Leclercq (2005) for a non-technical introduction].

First, we can classify them according to their look-back time and more precisely their space-time position relative to our actual position. This is summarized on Fig. 1 which represents our past-light cone, the location of the various systems (in terms of their redshift z) and the typical level at which they constrain the time variation of the fine structure constant. This systems include atomic clocks comparisons ($z = 0$), the Oklo phenomenon ($z \sim 0.14$), meteorite dating ($z \sim 0.43$), both having a space-time position along the world line of our system and not on our past-light cone, quasar absorption spectra ($z = 0.2 - 4$), cosmic microwave background (CMB) anisotropy ($z \sim 10^3$) and primordial nucleosynthesis (BBN, $z \sim 10^8$). Indeed higher redshift systems offer the possibility to set constraints on an larger time scale, but at the prize of usually involving other parameters such as the cosmological parameters. This is particularly the case of the cosmic microwave background and primordial nucleosynthesis, the interpretation of which requires a cosmological model.

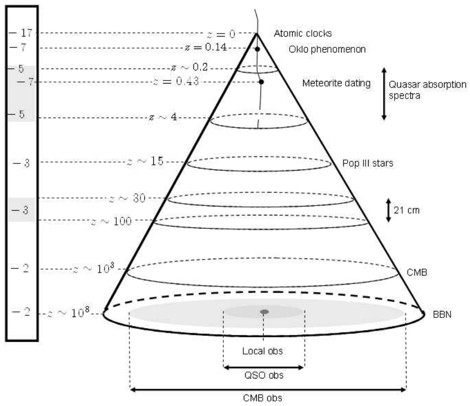

Fig. 1. Summary of the systems that have been used to probe the constancy of the fundamental constants and their position in a space-time diagram in which the cone represents our past light cone. The shaded areas represent the comoving space probed by different tests with respect to the largest scales probed by primordial nucleosynthesis.

The systems can also be classified in terms of the physics they involve in order to be interpreted (see Table 1). For instance, atomic clocks, quasar absorption spectra and the cosmic microwave background require only to use quantum electrodynamics to draw the primary constraints, so that these constraints will only involve the fine structure constant α, the ratio between the proton-to-electron mass ratio μ and the various gyromagnetic factors g_I. On the other hand, the Oklo phenomenon, meteorite dating and nucleosynthesis require nuclear physics and quantum chromodynamics to be interpreted.

2.2. Setting constraints

For any system, setting constraints goes through several steps that we sketch here.

First, any system allows us to derive an observational or experimental constraint on an observable quantity $O(G_k, X)$ which depends on a set of primary physical parameters G_k and a set of external parameters X, that usually are physical parameters that need to be measured or constrained (e.g. temperature,...). These external parameters are related to our knowledge

Table 1. Summary of the systems considered to set constraints on the variation of the fundamental constants. We summarize the observable quantities (see text for details), the primary constants used to interpret the data and the other hypotheses required for this interpretation. [α:fine structure constant; μ: electron-to-proton mass ratio; g_i: gyromagnetic factor; E_r: resonance energy of the samarium-149; λ: lifetime; B_D: deuterium binding energy; Q_{np}: neutron-proton mass difference; τ: neutron lifetime; m_e: mass of the electron; m_N: mass of the nucleon].

System	Observable	Primary constraints	Other hypothesis
Atomic clock	$\delta \ln \nu$	g_I, α, μ	-
Oklo phenomenon	isotopic ratio	E_r	geophysical model
Meteorite dating	isotopic ratio	λ	-
Quasar spectra	atomic spectra	g_p, μ, α	cloud properties
21 cm	T_b	g_p, μ, α	cosmological model
CMB	T	μ, α	cosmological model
BBN	light element abundances	$Q_{np}, \tau, m_e, m_N, \alpha, B_D$	cosmological model

of the physical system and the lack of their knowledge is usually referred to as systematic uncertainty.

From a physical model of the system, one can deduce the sensitivities of the observables to an independent variation of the primary physical parameters

$$\kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k}. \tag{2}$$

As an example, the ratio between various atomic transitions can be computed from quantum electrodynamics to deduce that the ratio of two hyperfine-structure transition depends only on g_I and α while the comparison of fine-structure and hyperfine-structure transitions depend on g_I, α and μ. For instance (Dzuba *et al.* (1999); Karshenboim(2005)) $\nu_{\mathrm{Cs}}/\nu_{\mathrm{Rb}} \propto \frac{g_{\mathrm{Cs}}}{g_{\mathrm{Rb}}}\alpha^{0.49}$ and $\nu_{\mathrm{Cs}}/\nu_{\mathrm{H}} \propto g_{\mathrm{Cs}}\mu\alpha^{2.83}$.

The primary parameters are usually not fundamental constants (e.g. the resonance energy of the samarium E_r for the Oklo phenomenon, the deuterium binding energy B_D for nucleosynthesis etc.) The second step is thus to relate the primary parameters to (a choice of) fundamental constants c_i. This would give a series of relations (see e.g. Müller *et al.* (2004))

$$\Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i. \tag{3}$$

The determination of the parameters d_{ki} requires first to choose the set of constants c_i (do we stop at the masses of the proton and neutron, or do we try to determine the dependencies on the quark masses, or on the Yukawa couplings and Higgs vacuum expectation value, etc.; see e.g. Dent *et al.*(2008) for various choices) and also requires to deal with nuclear physics and the intricate structure of QCD. In particular, the energy scales of QCD, Λ_{QCD}, is so dominant that at lowest order all parameters scales as Λ^n_{QCD} so that the variation of the strong interaction would not affect dimensionless parameters and one has to take the effect of the quark masses.

As an example, the Oklo phenomenon allows to draw a constraint on the value of the energy of the resonance. The observable O is a set of isotopic ratios that allow to reconstruct the average cross-sections for the nuclear network that involves the various isotopes of the samarium and gadolinium (this involves assumptions about the geometry of the reactor, its temperature that falls into X). It was argued (Damour and Dyson, 1996) on the basis of a model of the samarium nuclei that the energy of the resonance is mainly sensitive to α so that the only relevant parameter is $d_\alpha \sim -1.1 \times 10^7$. The level of the constraint $-0.9 \times 10^{-7} < \Delta\alpha/\alpha < 1.2 \times 10^{-7}$ that is inferred from the observation is thus related to the sensitivity d_α.

2.3. Unification and correlated variations

In the context of the unification of the fundamental interactions, it is expected that the variations of the various constants are not independent. By understanding these correlations we can set stronger constraints at the expense of being more model-dependent. We only illustrate this on the example of BBN, along the lines of Coc *et al.* (2007).

The abundances of nuclei synthesized during BBN rely on the balance between the expansion of the universe and the weak interaction rates which control the neutron to proton ratio at the onset of BBN (see Peter and Uzan (2005) for a textbook introduction). Basically, the abundance of helium-4 depends mainly on the neutron to proton ratio at the freeze-out time, $(n/p)_{\rm f} = \exp(-Q_{np}/kT_{\rm f})$, determined (roughly) by $G_F^2(kT_{\rm f})^5 = \sqrt{GN}(kT_{\rm f})^2$, N being the number of relativistic degrees of freedom; $Q_{np} = m_{\rm n} - m_{\rm p}$, G_F the Fermi constant and the neutron lifetime. It also depends on $t_{\rm N}$ the time after which the photon density becomes low enough for the photo-dissociation of the deuterium to be negligible. As a conclusion, the predictions of BBN involve a large number of parameters. In particular, $t_{\rm N}$ depends on the deuterium binding energy and on the photon-to-baryon ratio, η. Besides, one needs to include the effect of the fine structure constant in the Coulomb barriers (Bergström *et al.*, 1999). For a different analysis of the effect of varying fundamental constants on BBN predictions see e.g. Müller *et al.* (2004), Landau *et al.* (2006), Coc *et al.* (2007), Dent *et al.* (2007). Thus, the predictions are mainly dependent of the effective parameters $G_k = (G, \alpha, m_e, \tau, Q_{np}, B_D, \sigma_i)$ while the external parameters are mainly the cosmological parameters $X = (\eta, h, N_\nu, \Omega_i)$. It was found (Flambaum and Shuryak (2002), Coc *et al.* (2007): Fig. 3, Dent *et al.* (2007)) that the most sensitive parameter is the deuterium binding energy, B_D.

In a second step, the parameters G_k can be related to a smaller set of fundamental constants, namely the fine structure constant α, the Higgs VEV v, the Yukawa couplings h_i and the QCD scale $\Lambda_{\rm QCD}$ since $Q_{np} = m_n - m_p = \alpha a \Lambda_{\rm QCD} + (h_d - h_u)v$, $m_e = h_e v$, $\tau_n = G_F^2 m_e^5 f(Q/m_e)$ and $G_F = 1/\sqrt{2}v$. The deuterium binding energy can be expressed in terms of h_s, v and $\Lambda_{\rm QCD}$ (Flambaum and Shuryak, 2003) using a sigma nuclear model or in terms of the pion mass. Assuming that all Yukawa couplings vary similarly, the set of parameters G_k reduces to $\{\alpha, v, h, \Lambda_{\rm QCD}\}$ (again in units of the Planck mass). Several relations between these constants do exist. For instance, in grand-unified models the low-energy expression of $\Lambda_{\rm QCD}$, $\Lambda_{\rm QCD} = \mu\left(\frac{m_c m_b m_t}{\mu^3}\right)^{2/27}\exp\left[-\frac{2\pi}{9\alpha_3(\mu)}\right]$ for $\mu > m_t$ yields a relation between $\{\alpha, v, h, \Lambda_{\rm QCD}\}$ so that one actually has only 3 independent constants. Then, in all models in which the weak scale is determined by dimensional transmutation, changes in the Yukawa coupling h_t will trigger changes in v (Ibanez and Ross, 1982). In such cases, the Higgs VEV can be written as $v = M_p \exp\left[-\frac{8\pi^2 c}{h_t^2}\right]$, where c is a constant of order unity. I follows that we are left with only 2 independent constants. This number can even be reduced to 1 in the case where one assumes that the variation of the constants is trigger by an evolving dilaton (Damour and Polyakov (1994); Campbell and Olive (1995)). At each stage, one reduces the number of constants, and thus the level of the constraints, at the expense of some model dependence.

3. Theories with "varying constants"

3.1. Making a constant dynamical

The question of whether the constants of nature may be dynamical goes back to Dirac (1937) who expressed, in his "Large Number hypothesis", the opinion that very large (or small) dimensionless universal constants cannot be pure mathematical numbers and must not occur in the basic laws of physics. In particular, he stressed that the ratio between the gravitational and electromagnetic forces between a proton and an electron, $Gm_{\rm e}m_{\rm p}/e^2 \sim 10^{-40}$ is of the same order as the inverse of the age of the universe in atomic units, $e^2H_0/m_{\rm e}c^3$. He stated that these were not pure numerical coincidences but instead that these big numbers were not pure constants but reflected the state of our universe. This led him to postulate that

G varies[1] as the inverse of the cosmic time. Diracs' hypothesis is indeed not a theory and it was shown later (Jordan, 1937, 1939, Fierz, 1956) that a varying constant can be included in a Lagrangian formulation as a new dynamical degree of freedom so that one gets both a new dynamical equation of evolution for this degree of freedom and a modification of the other field equations with respect to their form derived under the hypothesis it was constant.

Let us illustrate this on the case of scalar-tensor theories, in which gravity is mediated not only by a massless spin-2 graviton but also by a spin-0 scalar field that couples universally to matter fields (this ensures the universality of free fall). In the Jordan frame, the action of the theory takes the form (Esposito-Farèse and Polarski, 2005)

$$S = \int \frac{\mathrm{d}^4 x}{16\pi G_*}\sqrt{-g}\left[F(\varphi)R - g^{\mu\nu}Z(\varphi)\varphi_{,\mu}\varphi_{,\nu} - 2U(\varphi)\right] + S_{\mathrm{matter}}[\psi; g_{\mu\nu}] \qquad (4)$$

where G_* is the bare gravitational constant. This action involves three arbitrary functions (F, Z and U) but only two are physical since there is still the possibility to redefine the scalar field. F needs to be positive to ensure that the graviton carries positive energy. In the Jordan frame, the matter is universally coupled to the metric so that the length and time as measured by laboratory apparatus are defined in this frame.

The action (4) defines an effective gravitational constant $G_{\mathrm{eff}} = G_*/F$. This constant does not correspond to the gravitational constant effectively measured in a Cavendish experiment. The Newton constant measured in this experiment is $G_{\mathrm{cav}} = G_* A_0^2(1+\alpha_0^2)$ where the first term, $G_* A_0^2$ corresponds to the exchange of a graviton while the second term $G_* A_0^2 \alpha_0^2$ is related to the long range scalar force. The gravitational constant depends on the scalar field and is thus dynamical.

The post-Newtonian parameters can be expressed in terms of the values of α and β today. For instance, $\gamma^{\mathrm{PPN}} - 1 = -\frac{2\alpha_0^2}{1+\alpha_0^2}$, The Solar system constraints imply α_0 to be very small, typically $\alpha_0^2 < 10^{-5}$ while β_0 can still be large. Binary pulsar observations (Esposito-Farèse, 2005) impose that $\beta_0 > -4.5$ and that $\dot{G}/G < 10^{-12}\,\mathrm{yr}^{-1}$.

Once such a model is specified, it can be further constrained by combining the bounds on the variation of the fundamental constants, the dynamics of the universe and its large scale structure (see e.g. Damour and Pichon (1999), Coc *et al.*(2006), Schimd *et al.*(2005), Martin *et al.*(2006), Riazuelo and Uzan (2002) for the particular case of scalar-tensor theories).

3.2. General dangers

Given the previous discussion, it seems a priori simple to cook up a theory that will describe a varying fine structure constant by coupling a scalar field to the electromagnetic Faraday tensor by including terms like $B(\phi)F_{\mu\nu}^2/4$ in the action so that the fine structure evolves according to $\alpha = B^{-1}$.

Such an simple implementation may however have dramatic implications. In particular, the contribution of the electromagnetic binding energy to the mass of any nucleus can be estimated by the Bethe-Weizäcker formula so that $m_{(A,Z)}(\phi) \supset 98.25\,\alpha(\phi)\,\frac{Z(Z-1)}{A^{1/3}}$ MeV. The sensitivity of the mass to a variation of the scalar field is expected to be of the order of

$$f_{(A,Z)} = \partial_\phi m_{(A,Z)}(\phi) \sim 10^{-2}\frac{Z(Z-1)}{A^{4/3}}\alpha'(\phi). \qquad (5)$$

It follows that the level of the violation of the universality of free fall is expected to be of the level of $\eta_{12} \sim 10^{-9}X(A_1,Z_1;A_2,Z_2)(\partial_\phi \ln B)_0^2$. Since the factor $X(A_1,Z_1;A_2,Z_2)$ typically ranges as $O(0.1-10)$, we deduce that $(\partial_\phi \ln B)_0$ has to be very small for the Solar system constraints to be satisfied. It follows that today the scalar field has to be very close to the minimum of the coupling function $\ln B$.

[1] Dirac hypothesis can also be achieved by assuming that e varies as $t^{1/2}$. Indeed this reflects a choice of units, either atomic or Planck units. There is however a difference: assuming that only G varies violates the strong equivalence principle while assuming a varying e results in a theory violating the Einstein equivalence principle. It does not mean we are detecting the variation of a dimensionful constant but simply that either $e^2/\hbar c$ or $Gm_e^2/\hbar c$ is varying.

Let us mention that such coupling terms naturally appear when compactifying a higher-dimensional theory (Peter and Uzan (2005), chapter 13) and in particular string theory. It is actually one of the definitive predictions for string theory that there exists a dilaton, that couples directly to matter (Taylor and Veneziano, 1988) and whose vacuum expectation value determines the string coupling constants (Witten, 1984).

For example, in type I superstring theory, the 10-dimensional dilaton couples differently to the gravitational and Yang-Mills sectors because the graviton is an excitation of closed strings while the Yang-Mills fields are excitations of open strings. For small values of the volume of the extra-dimensions, a T-duality makes the theory equivalent to a 10-dimensional theory with Yang-Mills fields localized on a D3-brane. When compactified on an orbifold, the gauge fields couple to fields M_i living only at these orbifold points with couplings c_i which are not universal. Typically, one gets that $M_4^2 = \mathrm{e}^{-2\Phi} V_6 M_I^8$ while $g_{YM}^{-2} = \mathrm{e}^{-2\Phi} V_6 M_I^6 + c_i M_i$. Unfortunately, the 4-dimensional effective couplings depend on the version of the string theory, on the compactification scheme and on the dilaton.

3.3. Ways out

While the tree-level predictions of string theory seem to be in contradiction with experimental constraints, many mechanisms can reconcile it with experiment. In particular, it has been claimed that quantum loop corrections to the tree-level action may modify the coupling function in such a way that it has a minimum (Damour and Polyakov, 1994). As explained in the former paragraph, the dilaton needs to be close to the minimum of the coupling function in order for the theory to be compatible with the universality of free fall. In the case of scalar-tensor theories, it was shown that when the coupling function enjoys such a minimum, the theory is naturally attracted toward general relativity (Damour and Nordtvedt, 1993). The same mechanism will apply if all the coupling functions have the same minimum. In that particular model the mass of any nuclei will typically be of the form $m_i(\phi) = \Lambda_{QCD}(\phi) \times \left(1 + a^q \frac{m_q}{\Lambda_{QCD}} + a^e \alpha\right)$, where a^q and are a^e are sensitivities. It follows that composition independent effects (i.e. $|\gamma^{\mathrm{PPN}} - 1|$, $|\beta^{\mathrm{PPN}} - 1|$, $\dot{G}/G$) and composition dependent effects (η, $\dot{\alpha}$, $\dot{\mu}$) will be of the same order of magnitude, dictated by the difference of the value of the dilaton today compared to its value at the minimum of the coupling function.

Another possibility is to invoke an environmental dependence, as e.g. the chameleon mechanism (Khoury and Weltman, 2004).

4. Conclusions

This short overwiew stresses the importance of the study of fundamental constant, and in particular of strong constraints on their variation at different epochs of the evolution of our universe, using different physical systems. These tests are particulary important with the necessity to test general relativity on astrophysical scales in order to better understand the dark sector of our cosmological model.

One question left without an answer is the one of what determines the value of the dimensionless constants. While the existing tests show that they have to be almost frozen since BBN time they do not give an answer to this question. By changing the value of these parameters, we change the physics and thus the properties of nature from the nuclear matter to the dynamics of the universe. It appears that the value of some constants has to be extremely tuned for a complex universe to develop (including e.g. complex structures such as nuclei, star and galaxies). These fine-tuning, to be distinguished from numerical coincidences, caracterizes some catastrophic boundaries in the space of fundamental constants across which some phenomena drastically change.

The recent developments of cosmology with the theory of inflation and the idea of the lanscape of string vacuaa has recently raised a growing interest in the idea that the value of (at least some of) the constants are enviromentally determined (see e.g. Hall and Nomura, 2007).

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