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Varying constants: constraints from seasonal variations

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Abstract. We analyse the constraints obtained from new atomic clock data on the possible time variation of the fine structure 'constant' and the electron-proton mass ratio and show how they are strengthened when the seasonal variation of Sun's gravitational field at the Earth's surface is taken into account.

1. Intoduction

General relativity and the standard model of particle physics depend on some 27 seemingly independent numerical parameters. These include the fine structure constants with determine the strengths of the different forces, matrix angles and phases and the relative masses of all known fundamental particles. These parameters are commonly referred to as the fundamental constants of Nature, although many modern proposals for fundamental physics predict that they are neither strictly fundamental nor constant.

Indeed, variation of the traditional 'constants', at some level, is a common prediction of most modern proposals for fundamental physics beyond the Standard Model. For instance, if the true fundamental theory exists in more than four space-time dimensions then the constants we observe are merely four-dimensional 'shadows' of the truly fundamental high dimensional constants. The four dimensional constants will then be seen to vary as the extra dimensions change slowly in size. Searches for and limits on the variations of the traditional constants therefore provide an important probe of fundamental physics.

2. Discussion

In many theoretical models of varying constants, one expects the values of the constants to evolve slowly as the Universe expands. Locally, this would manifest itself as a slow temporal drift in the values of the constants. Laboratory constraints on such a drift are generally found by comparing clocks based on different atomic frequency standards over a period of one or more years. The current best limit on the drift of the fine structure constant, α, was found by Rosenband et al. (2008). They repeatedly measured the ratio of aluminium and mercury single-ion optical clock frequencies, $f_{\mathrm{Al+}}/f_{\mathrm{Hg+}}$ over a period of about a year and found: $\dot{\alpha}/\alpha = (-5.3 \pm 7.9) \times 10^{-17}\,\mathrm{yr}^{-1}$.

If the 'constants', such as α, can vary, then in addition to a slow temporal drift one would also expect to see an annual modulation in their

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values (Shaw 2007). In many theories, the Sun perturbs the values of the constants by a factor roughly proportional to the Sun's Newtonian gravitational potential, which scales as the inverse of distance, r, between the Earth and the Sun. r fluctuates annually, reaching a minimum at perihelion in early January and a maximum at aphelion in July. The values of the constants, as measured here on Earth, should also therefore oscillate in a similar seasonal manner. Moreover, for many many cases, this seasonal fluctuation is predicted to dominate over any linear temporal drift (Shaw 2007; Barrow & Shaw 2008).

We suppose that the Sun creates a distance-dependent perturbation to the measured value of a coupling constant, C, of amplitude $\delta \ln C = C(r)$. If this coupling constant is measured on the surface of another body (e.g. the Earth) which orbits the first body along an elliptical path with semi-major axis a, period $T_{\rm p}$, and eccentricity $e \ll 1$, then to leading order in e, the annual fluctuation in C, $\delta C_{\rm annual}$ will be given by

$$\frac{\delta C_{\rm annual}}{C} = -c_C \cos\left(\frac{2\pi t}{T_{\rm p}}\right) + O(e^2), \qquad (1)$$

where $c_C \equiv e\, aC'(a)$, $C'(a) = dC(r)/dr|_{r=a}$ and $t = nT_{\rm p}$, for any integer n, corresponds to the moment of closest approach (perihelion); here $a = 149,597,887.5$ km is the semi-major axis of the Earth's orbit. In the case of the Earth moving around the Sun, over a period of 6 months from perihelion to aphelion one would therefore measure a change in the constant C equal to $2c_C$. As we stated above, in many theoretical modes,$\delta \ln C = C(r) \propto \Delta U_{\odot}(r)$ where $U_{\odot}(r) = -GM/r$ is the Newtonian potential of the Sun. We introduce sensitivity parameters k_C defined by $\delta \ln C = k_C \Delta U_{\odot}$. Hence:

$$c_C = ea\frac{GM_{\odot}}{a^2}k_C = 1.65 \times 10^{-10} k_C. \qquad (2)$$

Once one specifies a theoretical model of varying constants, the k_C are determined. hence measuring or constraining the k_C limits the underlying theory. One generally expects $k_C \sim O(1)$ in higher dimensional theories such as String Theory, however, as we shall see, observations generally constrain $k_C \ll 1$.

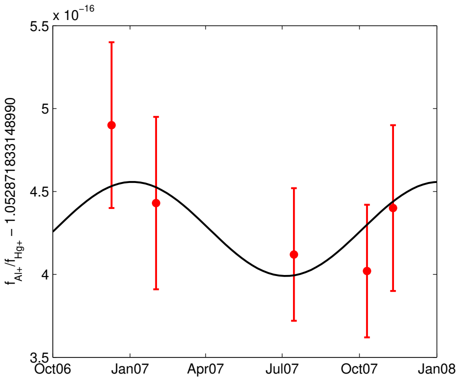

Fig. 1. Frequency ratio $f_{\rm Al+}/f_{\rm Hg+}$ as measured by Rosenband et al. (2008). The solid black line shows the maximum likelihood fit for a seasonal variation.

Rosenband et al. (2008) fitted a linear drift in α to their data finding $\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17}\,{\rm yr}^{-1}$. We fitted the expected form of any annual fluctuation, Eq. 1, to the measured values of $f_{\rm Al+}/f_{\rm Hg+}$(Barrow & Shaw 2008). Fig. 1 shows the best-fit line superimposed on the (Rosenband et al. 2008) data. It should be noted that the magnitudes of the systematic errors for the middle three data points were not verified to the same precision as they were from the first and last data points (Rosenband et al. 2008). We do not expect to this have a great effect on the resulting constraint on k_α. Using $\delta \ln(f_{\rm Al+}/f_{\rm Hg+} = (3.19 + 0.008)\delta\alpha/\alpha$, (Rosenband et al. 2008), a maximum likelihood fit to the data gives

$$k_\alpha = (-5.4 \pm 5.1) \times 10^{-8}. \qquad (3)$$

This limit on k_α is almost an 2 order of magnitude improvement on the previous limit of $k_\alpha = (2.5 \pm 3.1) \times 10^{-6}$ from by Blatt et al. (2008).

The frequency shifts measured by Rosenband et al. (2008) are not sensitive to changes in the electron-proton mass ratio: $\mu = m_{\rm e}/m_{\rm p}$. However, measurements of optical transition frequencies relative to Cs are sensitive to both μ and α, and H-maser atomic clocks can detect changes in the light quark to proton mass ratio: $q = m_{\rm q}/m_{\rm p}$. We define sensitivity parameters k_μ and k_q for μ

and q respectively. We then find that the Yb^+ frequency measurements of Peik et al. (2004) give $k_\alpha + 0.51k_\mu = (7.1 \pm 3.4) \times 10^{-6}$. Combining this limit with that on k_α and the limits $k_\alpha + 0.36k_\mu = (-2.1 \pm 3.2) \times 10^{-6}$ (Blatt et al. 2008), $k_\alpha + 0.17k_\mu = (3.5 \pm 6.0) \times 10^{-7}$ (Fortier et al. 2007) and $k_\alpha + 0.13k_\mathrm{q} = (1 \pm 17) \times 10^{-7}$ Ashby et al. (2007), we find (Barrow & Shaw 2008)

$$k_\mu = (3.9 \pm 3.1) \times 10^{-6}, \tag{4}$$

$$k_\mathrm{q} = (0.1 \pm 1.4) \times 10^{-5}. \tag{5}$$

Again these represent improvements on the previous limits of $k_\mu = (-1.3 \pm 1.7) \times 10^{-5}$ and $k_\mathrm{q} = (-1.9 \pm 2.7) \times 10^{-5}$ found by Blatt et al. (2008).

Seasonal fluctuations are predicted by a varying constant theories because the constants depend on the the vacuum expectation value one or more scalar fields, ϕ_I, which interact with normal matter. In the solar system the field equations of these scalars typically reduce to Poisson equations. We label the different constants $C_j(\phi_I)$ and then:

$$\nabla^2\phi_I \approx 4\pi G \sum_j \frac{\partial \ln C_j}{\partial \phi_I}\frac{\delta\rho}{\delta \ln C_j}. \tag{6}$$

We define $\beta_{Ij} = \partial \ln C_j / \partial \phi_I$ and then for small variations of the scalar fields and the constants we have:

$$\nabla^2 \ln C_i \approx \left(\sum_{I,j} \beta_{Ii}\beta_{Ij}\frac{\delta \ln \rho}{\delta \ln C_j}\right) 4\pi G\rho. \tag{7}$$

It follows that:

$$||_{C_i} = \left(\sum_{I,j} \beta_{Ii}\beta_{Ij}\frac{\delta \ln \rho_\odot}{\delta \ln C_j}\right).$$

We define $\lambda_j^{(\odot)} = \delta \ln \rho / \delta \ln C_j$ which can be calculated from the Standard Model. Measurements of $||_{C_i}$ from seasonal variations therefore determine the model parameters $\sum_I \beta_{Ii}\beta_{Ij}$.

The presence of spatial gradients in the scalar fields with couple to normal matter also results in new or fifth' forces. The magnitude of the new force towards the Sun on a test body with density ρ_t, mass m_t at a distance r from the Sun is given by:

$$F_\phi = m_t\beta_{Ij}\lambda_j^{(t)}\frac{\partial\phi_I}{\partial r} = \frac{Gm_tM_\odot}{r}\sum_j k_{C_j}\lambda_j^{(t)}. \tag{8}$$

Now F_ϕ depends on $\lambda_j^{(t)}$ which depends on the composition of the test mass. This composition dependence violates the universality of free-fall and hence the weak equivalence principle (WEP). WEP violations searches measure the differential acceleration of two differently composes test masses towards the Sun which is proportional to $\sum_i k_{C_i}\delta\lambda_i^{(t)}$; $\delta\gamma_i^{(t)}$ is the difference between the λ_j values for the two test masses. Hence WEP violation searches indirectly probe the k_{C_i}. The magnitude of any composition-dependent fifth force toward the Sun is currently constrained to be no stronger than about few $\times 10^{-13}$ times than the gravitational force (Schlamminger et al. 2008). To extract limits on the individual k_{C_i} one must accurately calculate the λ_i parameters for the test masses and have limits from different experiments to eliminate the degeneracy between the different k_{C_i}.

The constraints from WEP tests indirectly bound k_α. Indeed, they often provide the tightest constraints on k_α (Shaw 2007; Barrow & Shaw 2008; **?**). A recent and thorough analysis of the WEP violation constraints on k_α (**?**) found:

$$|k_\alpha| \lesssim (0.23 - 1.4) \times 10^{-8}$$

with a similar limit on k_q. The uncertainty in this limit is to due to how one models nuclear structure. Despite these uncertainties, it is clear that WEP violation constraints from laboratory experiments currently provide the strongest, albeit indirect, bounds on k_α and the other k_{C_i}.

We now consider the sensitivity that would be required of an atomic clock to provide tighter constraints on k_α than those coming indirectly from limits on WEP violation. Suppose that the ratio of two transition frequencies, $f_\mathrm{A}/f_\mathrm{B}$, can be measured with a sensitivity σ_f, and that $\delta \ln(f_\mathrm{A}/f_\mathrm{B}) = S_\alpha \delta\alpha/\alpha$ (typically $S_\alpha \sim \mathcal{O}(1)$, although some transitions exhibit a greatly increased sensitivity (Flambaum

2006)). The sensitivity to changes in α is then given by $\sigma_\alpha = \sigma_f / S_\alpha$. By simulating data sets, we found that the sensitivity to k_α is significantly improved if one makes $N_m \gtrsim 12$ measurements per year (at roughly regular intervals). With $N_m \gtrsim 12$, by performing a bootstrap linear regression with 10^5 re-samplings of the simulated data points, we find that the sensitivity, σ_k to k_α is roughly:

$$\sigma_k \approx 0.69 \times 10^{10} \frac{\sigma_\alpha}{\sqrt{N_y(N_m - 1)}}.$$

where N_y is the number of years for which data is taken. The total number of measurements is therefore $N_y N_m$. Indirect constraints currently have a sensitivity no better than $\sigma_k = 2.3\times10^{-9}$ (**?**). This would be surpassed by direct measurements if $\sigma_\alpha < 2.5\sqrt{N_y(N_m - 1)} \times 10^{-19}$. For example, if we take measurements every 20 days (or so) over a single year ($N_m = 18$, $N_y = 1$) then we would require $\sigma_\alpha \lesssim 10^{-18}$. Flambaum (2006) have noted that first excited states in the ^{229}Th nucleus is particularly sensitive to changes in α and μ with $S_\alpha \sim 10^5$, allowing for a $\sigma_\alpha \lesssim 10^{-20}$. Such a sensitivity would limit k_α at the 10^{-11} level – two orders of magnitude better than the current WEP violation constraints. In summary: we have shown how new laboratory constraints on possible time variation in the fine structure 'constant' and the electron-proton mass ratio can yield more sensitive limits by incorporating the effects of the seasonal variation of the Sun's gravitational field at the Earth's surface. This seasonal variation is expected in all theories which require that the covariant d'Alembertian of any scalar field driving variation of a 'constant' is proportional to the dominant local source of gravitational potential. The recent experimental results from (Rosenband et al. 2008) and (Peik et al. 2004) have reached the sensitivity of the quasar observations of varying α and μ made at high redshift and we have shown may soon provide stronger bounds on varying constants than conventional ground-based WEP experiments.

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