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SNIa, white dwarfs and the variation of the gravitational constant

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Abstract. White dwarfs represent the last evolutionary stage of stars with masses smaller than $10 \pm 2\, M_{\odot}$. Since their mechanical structure is sustained by the pressure of degenerate electrons, they do not radiate nuclear energy and their evolution is just a simple gravothermal process of cooling. The only exception to this is the case of white dwarfs in close binary systems where, by accretion of matter from the companion, they can reach the Chandrasekhar's mass and explode as a Type Ia supernova (SNIa). The cooling of single white dwarfs and the properties of SNIa strongly depend on the precise value of G and on its possible secular variation. Consequently, white dwarfs can be used to constrain such hypothetical variations. Although the bounds obtained in this way have been currently superseeded by other more accurate methods, when the ongoing surveys searching for SNIa and white dwarfs will be completed, the expected bounds will be as tight as $\sim 10^{-13}$ yr^{-1}.

Key words. Stars: white dwarfs — stars: supernovae — gravitation

1. Introduction

All the laws of the Nature that relate different dynamical characteristics contain parameters that are assumed to be independent of the space-time location and are regarded as *constants of nature*. The gravitation constant, G, is an example. The critical role that G plays in the gravitation theory and the possibility, introduced by theories that unify gravity with other interactions, that G could vary in time and space have bursted the interest in detecting such variations or, at least, to bound them as tight as possible.

The newtonian constant is very difficult to measure (in fact, what we know with precision is the product GM) and, as a consequence, the accuracy with which we are able to measure G is the poorest. To see this, it is enough to compare the precision of the measured value of $G = 6.673(10) \times 10^{-11}$ m^3 kg^{-1} s^{-1} with, for instance, the measured value of the Planck constant, $\hbar = 1.0554571596(82) \times 10^{-34}$ J s. Because of these difficulties, a large variety of astronomical techniques have been used Uzan (2003); García–Berro et al (2007) to detect or,

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at least, to bound the variations of G. In any case, the uncertainties and model dependences are so large that is wise to use different astrophysical techniques to constrain or verify theories involving a rolling G. Here, we show why white dwarfs are excellent laboratories to test possible variations of G. We advance three reasons for this: i) their evolution is just a simple process of cooling, ii) the basic physical ingredients necessary to predict their evolution are well identified, although not necessarily well understood, and iii) there is an impressively solid observational background to which we can compare the results. In the rest of this paper we describe the methods that can be used to achieve this goal. In principle, there are two independent methods to relate the properties of white dwarfs with the changes of G: using the Chandrasekhar's mass and its role in the luminosity of SNIa explosions and using the characteristic cooling time of single white dwarfs.

2. The cooling rate of white dwarfs

When white dwarfs are cool enough, their luminosity has entirely a gravothermal origin. Any change of G modifies the energy balance of the interior and, consequently, also modifies their luminosity. Formally, the influence of a secular variation of G can be expressed as:

$$L = -\frac{dB}{dt} + \Omega \frac{1}{G}\frac{dG}{dt} \qquad (1)$$

where $B = U + \Omega$ is the total binding energy, U is the total inernal energy and Ω is the gravitational energy García–Berro et al (1995). It is important to realize here that the internal energy is stored in the form of chemical potential (roughly the Fermi energy of electrons) and thermal kinetic energy, for which reason only the second term contributes to the luminosity in the case of a varying G.

The first attempt to obtain constraints on $\dot{G}$ from the cooling of white dwarfs Vila (1976) was unsuccessful due to the lack of reliable observational data and the uncertainties in the theoretical models. Fortunately, this situation has changed nowadays and it is possible to obtain precise information from the white dwarf luminosity function and from the secular drift of the pulsation period.

2.1. The white dwarf luminosity function

The white dwarf luminosity function is defined as the number of white dwarfs of a given luminosity per unit of magnitude interval, and can be computed as:

$$n(l) \propto \int_{M_{\rm i}}^{M_{\rm s}} \Phi(M)\,\Psi(\tau)\tau_{\rm cool}(l, M)\,dM \qquad (2)$$

where l is the logarithm of the luminosity in solar units, M is the mass of the parent star (for convenience all white dwarfs are labeled with the mass of the main sequence progenitor), $t_{\rm cool}$ is the cooling time down to luminosity l, $\tau_{\rm cool} = dt/dM_{\rm bol}$ is the characteristic cooling time, $M_{\rm s}$ and $M_{\rm i}$ are the maximum and the minimum masses of main sequence stars able to produce a white dwarf of luminosity l, $t_{\rm PS}$ is the lifetime of the progenitor of the white dwarf, and T is the age of the population under study. The remaining quantities, the initial mass function, $\Phi(M)$, and the star formation rate, $\Psi(t)$, are not known a priori and depend on the astronomical properties of the stellar population under study. The observed cut-off of the white dwarf luminosity function produced by the finite age of the Galaxy provides a bound to a variation of G. This bound was set to be García–Berro et al (1995):

$$-(1 \pm 1) \times 10^{-11}\,\mathrm{yr}^{-1} < \frac{\dot{G}}{G} < 0$$

These values will be noticeably improved as soon as new surveys, like the SDSS, or the data coming from astronomical observatories like *Gaia* or *SIM*, will effectively constrain the number of white dwarfs with very low luminosities, populating the dimmest bins of the white dwarf luminosity function.

2.2. Pulsating white dwarfs

During their cooling, white dwarfs cross some specific regions of the Hertzsprung-Russell diagram where they become unstable and pulsate. The multiperiodic character of the pulsations and the value of the periods (100 s to

1000 s), much larger than the period of radial pulsations (∼ 10 s), indicate that white dwarfs are g-mode pulsators (the restoring force is gravity). As variable white dwarfs cool down, the oscillation period, P, changes as a consequence of the changes in the mechanical structure. This secular drift can be well approximated by Winget et al. (1983):

$$\frac{\dot{P}}{P} \simeq -a\frac{\dot{T}}{T} + b\frac{\dot{R}}{R} \tag{3}$$

where a and b are positive constants of the order of unity. The first term of the right hand side reflects the fact that as the star cools down the degree of degeneracy increases, the Brunt-Väisälä frequency decreases and, as a consequence, the oscillation period increases. At the same time, the residual gravitational contraction, that is always present, tends to decrease the oscillation period. The later effect is accounted for by the last term in Eq. (3).

There are three types of variable white dwarfs, the DOV, the DBV and the DAV white dwarfs. In the case of the hottest ones, the DOV, the gravitational contraction is still significant and the second term in Eq. (3) is not negligible. In fact, for these stars $\dot{P}$ can be positive or negative depending on the character of the oscillation mode. If the mode is dominated by the deep regions of the star, $\dot{P}$ is positive. If the mode is dominated by the outer layers, $\dot{P}$ is negative. The secular drift of the prototype of this kind of stars (PG1159) is $\dot{P} = (13.07 \pm 0.3) \times 10^{-11}$ s/s for its 516 s pulsation period. However, the uncertainties in the modelling of their interior have prevented obtaining reliable conclusions until now.

The DBV stars are characterized by the lack of the hydrogen layer in their envelopes and by effective temperatures of the order of $T_{\rm eff} \sim 25,000$ K. Since they are cooler than DOV white dwarfs, the radial term is negligible and $\dot{P}$ is always positive. The expected drift ranges from $\dot{P} \sim 10^{-13}$ to 10^{-14} s/s Córsico & Althaus (2004), but it has not been yet measured with enough accuracy.

The DAV white dwarfs, or ZZ Ceti stars, are characterized by the presence of a very thin atmospheric layer made of pure hydrogen and by effective temperatures ranging from $T_{\rm eff} \sim 12,000$ to $15,000$ K. As in the case of DBV stars, they are so cool that the radial term is negligible and $\dot{P}$ is always positive. The drift has been measured for the $P = 215.2$ s mode of G117–B15A (Kepler et al. 2005), $\dot{P} = (3.57 \pm 0.82) \times 10^{-15}$ s/s. The seismological analysis of G117–B15A indicates that the best fit is obtained for $M_* = 0.55\,M_\odot$, $\log M_{\rm He}/M_* = -2$ and $\log M_{\rm H}/M_* = -4.04$ Córsico et al. (2001), that gives a cooling rate of $\dot{P} = 3.9 \times 10^{-15}$ s/s, very similar to the observational one. This secular drift provides a 2σ bound Benvenuto et al (2004):

$$-2.5 \times 10^{-10}\,{\rm yr}^{-1} < \frac{\dot{G}}{G} < 4.0 \times 10^{-11}\,{\rm yr}^{-1}$$

Once more, an improvement of both the seismological models and of the accuracy to which the drift of the observed periods is measured can result in significantly better bounds in the next future. Although, given the weaker dependence of the drift of the period of pulsation on G, the bounds obtained in this way will be less tight than those obtained from the luminosity function.

3. Type Ia supernova

Type Ia supernovae are thought to be the result of the thermonuclear explosion of a carbon-oxygen white dwarf with a mass near Chandrasekhar's limit in a close binary system. Light curve models show that the peak luminosity is proportional to the mass of nickel synthesized which,in turn, is probably a fixed fraction of the mass of the exploding star ($M_{\rm Ni} \propto M_{\rm Ch}$), which depends on the gravitation constant as

$$M_{\rm Ch} = \frac{(\hbar c)^{3/2}}{m_{\rm p} G^{3/2}} \tag{4}$$

The actual fraction depends on the model considered Gómez–Gomar et al (1998), but the physical mechanisms relevant to SNIa naturally relate the energy yield to the Chandrasekhar's mass Gaztanaga et al (2002). The apparent magnitude of SNIa at the peak luminosity is then

$$m(z) = M_0 + \frac{15}{4}\log(\frac{G}{G_0}) + 5\log d_{\rm L} + 25 \tag{5}$$

where M_0 is the absolute magnitude at maximum, $d_L(z, \Omega_M, \Omega_\Lambda, G)$ is the luminosity distance and G_0 is the present value of the gravitation constant. The variation of G affects both the luminosity distance and the Chandrasekhar's mass, but it has been argued that the first dependence can be ommitted Gaztanaga et al (2002); Riazuelo and Uzan (2002). As an exemple, for the existing data in the epoch, Lorén–Aguilar et al (2003) obtained for the currently favored cosmological scenario:

$$-1.4 \times 10^{-11}\,\mathrm{yr}^{-1} < \frac{\dot{G}}{G} < +2.6 \times 10^{-11}\,\mathrm{yr}^{-1}$$

This bound is modest when compared with the current values obtained from lunar laser ranging methods, but using more complete samples this method will allow to put constraints at differents cosmological epochs, a key feature that other local methods cannot offer.

4. Conclusions

Because of their simplicity, white dwarfs can be considered excellent complementary laboratories for testing new physics. The high precision luminosity functions that are progressively becoming available and the pulsation drift of degenerate variables can be used to provide independent new bounds to any possible variation of G. Type Ia supernovae also turn out to be excellent targets to check whether the gravitational constant has experienced a variation at cosmological epochs.

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