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The breaking of the equivalence principle in theories with varying α

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Abstract. Bekenstein has shown that violation of Weak Equivalence Principle is strongly supressed in his model of charge variation. In this paper, it is shown that nuclear magnetic energy is large enough to produce observable effects in Eötvös experiments.

1. Introduction

An interesting possibility is that a space variation of fundamental constants should produce a violation of the Weak Equivalence Principle Misner et al. (2000), a fact that can be proved easily using energy conservation Haugan (1979). The most sensitive forms of those tests are the *Eötvös experiments*. Several accurate tests have been carried in the second half of the 20th century and up to now. These tests impose strict bounds on parameters describing WEP violations Will (2000); Chamoun et al. (1999).

However, in his 2002 paper Bekenstein Bekenstein (2002) proved that a violation of WEP is highly unlikely in his model. We shall discuss briefly this issue later on, but the origin of this statement is a wonderful cancellation of electrostatic sources of the ψ field, leading to a null effect in the lowest order. No such cancellation happens for magnetostatic contribution, but a simple examination of the Solar System magnetic energy density suggests that a breakdown of WEP should be inobservable. Here, we discuss the detection of a space variation of α in Bekenstein's model, considering the fluctuations of magnetic fields in quantum systems.

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2. A survey of Bekenstein model

Bekenstein's proposal Bekenstein (1982, 2002) was to modify Maxwell's electromagnetic theory introducing a field $\epsilon = e^{\psi}$ to describe α variation.

$$e(x^{\mu}) = e_0 e^{\psi(x^{\mu})} \qquad \alpha(x^{\mu}) = e^{2\psi(x^{\mu})}\alpha_0 \tag{1}$$

where e_0, α_0 are reference values of the electric charge and α. The general equations of motion in Bekenstein's theory are

$$\left(e^{-\psi}F^{\mu\nu}\right)_{,\nu} = 4\pi j^{\mu}, \tag{2a}$$

$$j^{\mu} \equiv e_0 c v^{\mu}\frac{\delta^3(\boldsymbol{x} - z(\tau))}{\gamma\sqrt{-g}}, \tag{2b}$$

$$\Box\psi = \frac{l_B^2}{\hbar c}\left(\frac{\partial\sigma}{\partial\psi} - \frac{F^{\mu\nu}F_{\mu\nu}}{8\pi}\right), \tag{2c}$$

$$\sigma = \sum mc^2\gamma^{-1}(-g)^{-1/2}\delta^3[\boldsymbol{x} - z(\tau)], \tag{2d}$$

the latter quantity being the rest mass energy density.

In his papers Bekenstein uses an ensemble of classical particles to represent matter. This is not a good model of matter wherever quantum phenomena are important, neither at high energy scales or small distances scales.

From the above equations of motion he Bekenstein (2002) derives several statements.

Cancellation statement For a electrostatic field equation (2c), in the source term for ψ the first term cancels almost exactly the second and the asymptotic value of ψ is almost exactly suppressed.

WEP for electric charges The equation of motion of a system of charges in an electric field, in the limit of very small velocities, reduces to

$$M\ddot{\boldsymbol{Z}} = Q\boldsymbol{E} \tag{3}$$

and where M and Q are the total mass and charge of the system. Thus, there is no WEP violation.

WEP for magnetic dipoles In the equation of motion for ψ for this kind of static systems, there is no cancellation of sources. From an estimate of the field intensities in the Solar System, Bekenstein states that no observable WEP violation can be detected in laboratory experiments.

3. Motion of a composite body in the ψ field

We are describing the Lagrangian of a body composed of point-like charges, such as an atom or an atomic nucleus. We shall work in the nonrelativistic limit for the charges, but we shall keep for the moment the full expression for the electromagnetic field. We shall treat the system as classical and later on quantize it in a simple way. The techniques we use are from the $TH\epsilon\mu$ formalism Light-Lee (1973); Will (2000). We assume that there are external dilaton ψ and Newtonian gravitationa ϕ_N fields acting over the body, but we shall neglect the self fields generated.

For a macroscopic solid body we shall be interested in the motion of the center of mass.

$$\begin{aligned} L = &- M_{\rm tot}\left[c^2 - \frac{V_{\rm CM}^2}{2} - \phi_N(\boldsymbol{R}_{\rm CM}) + \right] \\ &+ 2\psi(\boldsymbol{R}_{\rm CM})E_m + \ldots \end{aligned} \tag{4}$$

The electrostatic contribution cancels with the mass dependence on ψ, according to Bekenstein theorem, and the neglected terms are of either tidal order, negligible in laboratory tests of WEP, or of higher order in ψ (paper in production).

The above Lagrangian shows that a body immersed in external gravitational and Bekenstein fields will suffer an acceleration

$$\ddot{\boldsymbol{R}}_{\rm CM} = \boldsymbol{a} = \boldsymbol{g} + 2\frac{E_m}{M}\,\boldsymbol{\nabla}\psi|_{\rm CM}\,. \tag{5}$$

The latter term is the anomalous acceleration generated by the Bekenstein field. The acceleration difference (5) is tested in Eötvös experiments.

4. Magnetic energy of matter

In a quantum model of matter, magnetic fields originate in not only in the stationary electric currents that charged particle originate and their static magnetic moments but also in quantum fluctuations of the number density. These contributions to the magnetic energy have been computed in Ref. Haugan-Will (1977); Will (2000) from a minimal nuclear shell model. Since the magnetic energy density is concentrated near atomic nuclei, it can be represented in the form

$$e_m(\boldsymbol{x}) \simeq \sum_b E_m^b n_b(\boldsymbol{x}) \qquad \zeta_m^b = \frac{E_m^b}{M_b c^2} \tag{6}$$

where index b runs over different nuclear species and ζ_m^b is the fractional contribution of the magnetic energy to rest mass. Then

$$e_m(\boldsymbol{x}) = \bar{\zeta}_m(\boldsymbol{x})\rho(\boldsymbol{x})c^2 \tag{7a}$$

$$\bar{\zeta}_m(\boldsymbol{x}) = \frac{\sum_b \zeta_b\rho_b(\boldsymbol{x})}{\rho(\boldsymbol{x})} \tag{7b}$$

where $\rho(\boldsymbol{x})$ is the local mass density. With expression (7) we can write the equation for the source due to the magnetic contribution in the form

$$\nabla^2\psi = -8\pi\kappa^2 c^2 e^{-2\psi}\bar{\zeta}_m\rho. \qquad (8)$$

For small ψ we can find a solution for an arbitrary distribution of sources whose asymptotic behaviour can be expressed in terms of the newtonian gravitational potential

$$\begin{aligned}\psi =& \frac{8\pi\kappa^2}{GM}\phi_N(r)\int_0^\infty x^2\bar{\zeta}_m(x)\rho(x)dx \\ =& \, 2\left(\frac{l_B}{l_P}\right)^2 \tilde{\zeta}_m\frac{\phi_N(r)}{c^2},\end{aligned} \qquad (9)$$

where $\tilde{\zeta}_m$ is the mass averaged value of ζ_m and we have introduced the Planck length l_P.

5. Results and conclusions

From (9) we can obtain for the differential acceleration of a pair of different bodies. Then, using the most accurate versions of the Eötvös experiment yields,

$$\left(\frac{l_B}{l_P}\right)^2 = 0.0003 \pm 0.0006 \qquad (10)$$

from which we get the "3σ" upper bound

$$\left(\frac{l_B}{l_P}\right)^2 < 0.002 \qquad \frac{l_B}{l_P} < 0.05 \qquad (11)$$

It is easy to deduce that strict upper bounds can be set from Eötvös experiments on the Bekenstein parameter l_B/l_P even if the electrostatic field does not generate ψ field. These bounds are much larger than the ones that would result if electrostatic energy density would generate ψ field intensity. This calculation was carried in the 1982 paper of Bekenstein Bekenstein (1982) and has been repeated several times (e.g. Dvali-Zaldarriaga (2001); Chamoun et al. (1999); Mosquera et al. (2008)) with the result

$$\left(\frac{l_B}{l_P}\right)_{\rm el} < 8.7 \times 10^{-3}, \qquad (12)$$

one order of magnitude smaller than (11). It is interesting to compare our result (11) with the results obtained from an analysis of all evidence from time variation of the fine structure constant α Mosquera et al. (2008). In that paper, an effective value of $\zeta = 10^{-4}$ was used, following the suggestion of ref. Sandvik et al. (2000) and a 1σ bound on $(l_B/l_P)^2 < 0.003$ was found. From the estimate of $\zeta_{\rm H}$ in reference Bekenstein (2002) we compute an effective value of $\zeta_U = 2.7 \times 10^{-5}\Omega_B \simeq 1.4 \times 10^{-6}$ and so we find a 3σ upper bound

$$\frac{l_B}{l_P} < 0.8 \qquad (13)$$

one order of magnitude larger than (11).

In conclusion, we have shown that very strict bound can be put on the Bekenstein model parameter l_B/l_P from the quantum fluctuations of the magnetic fields of matter. From equation (11) one should discard the Bekenstein model, but since it can be obtained as a low energy limit of string models, the latter conclusion should be taken with a grain of salt.

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