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Magnetic acceleration of relativistic jets

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Abstract. This is a brief review of the recent developments in the theory of magnetic acceleration of relativistic jets. We attempt to explain the key results of this complex theory using basic physical arguments and simple calculations. The main focus is on the standard model, which describes steady-state axisymmetric ideal MHD flows. We argue that this model is over-restrictive and discuss various alternatives.

Key words. Magnetohydrodynamics – Relativistic processes – Galaxies: jets – Methods: analytical – Methods: numerical – Gamma-Rays: stars

1. Introduction

Collimated flows, or jets, are observed in a variety of astrophysical systems but the most spectacular examples are related to disk accretion onto a compact central object. This suggests that disk accretion is an essential element of cosmic jet engines, via providing a source of power or collimation, or both. The most remarkable property of astrophysical jets is that their length exceeds the size of the compact object, and hence the size of the central engine, by many orders of magnitude. For example, AGN jets are generated on the scale no more than hundred gravitational radii of the central black hole, $\sim 10^{15}$cm, and propagate up to the distance of $\sim 10^{24}$cm, where they create extended radio lobes. Along the jets, the specific volume of plasma increases enormously and the corresponding adiabatic losses, in combination with various radiative losses, ensure that the plasma particles loose essentially all their "thermal" energy, which they might have had inside the central engine, very quickly. Yet, the observations show that the jet brightness does not decline so rapidly. This suggests that most of the jet energy is in a different form and that the observed emission is the results of its slow dissipation.

One possibility is that astrophysical jets are supersonic, kinetic energy-dominated flows (Scheuer 1974; Blandford & Rees 1974). Indeed, a number of factors make the idea very attractive. First, such flows do not require external support in order to preserve their collimation. Second, they are much more stable and can propagate large distances without significant energy losses, in an essentially ballistic regime. Third, when they interact with the external medium the result is shocks, which dissipate kinetic energy locally and thus can produce bright compact emission sites, reminiscent of the knots and hot spots of astrophysical jets.

Generic industrial jet engine consists of a chamber, which is being filled with very hot gas, and a carefully designed nozzle through which the gas escapes with a supersonic speed. The acceleration mechanism is thermal – it is the thermodynamic pressure force that accelerates this gas and convert its thermal energy into the bulk motion kinetic energy. Apparently, a very similar configuration may form naturally during the gravitational collapse of rotating massive stars. It has been shown that, for a sufficiently rapid rotation, the centrifugal force eventually "evacuates" the polar region around the central black hole. The neutrinos, emitted from its super-critical accretion disk, can then fill the cavity with ultra-relativistically hot plasma via neutrino-antineutrino annihilation. This plasma can then expand in the direction of least resistance, which is obviously the polar direction, and thus create a collimating nozzle. In fact, this may the origin of jets associated with long GRBs (MacFadyen & Woosley 1999).

Similar ideas have been explored earlier for ANG jets but the results were rather discouraging, which stimulated search for alternatives. Gradually, the magnetic model emerged, where the Maxwell stresses were responsible for powering, acceleration, and even collimation of jets. In principle, this model even allows to avoid the kinetic energy-dominated phase as the observed emission can be powered via dissipation of magnetic energy (e.g. Lyutikov & Blandford 2003). However, thanks to the success of the shock model in many areas of astrophysics, the current magnetic paradigm assumes that most of the jet Poynting flux is first converted into bulk kinetic energy and then dissipated at shocks (shocks in highly magnetised plasma are inefficient).

2. The standard model

The problem of magnetic acceleration is rich and mathematically complex. This complexity is the reason why most theoretical efforts have been directed towards development of the *standard model*, which deals with steady-state axisymmetric flows of perfect fluid. Although this is the simplest case it is still impossible to give a comprehensive and mathematically rigorous review within the scope of this presentation. Instead, I will focus more on physical arguments.

2.1. Acceleration in supersonic regime

One clear difference between non-relativistic and relativistic flows is that in the non-relativistic limit most of the energy conversion occurs already in the subsonic regime whereas in the relativistic limit this occurs in the supersonic regime. This is true both for the thermal and magnetic mechanisms

Consider first the non-relativistic limit. For a hot gas with polytropic equation of state, the thermal energy density $e = p/(\gamma - 1)$ and the sound speed $a_s^2 = \gamma p/\rho$, where γ is the ratio of specific heats, p and ρ are the gas pressure and density respectively. From this we immediately find that when the flow speed equals to the sound speed $\rho v^2 = \gamma(\gamma - 1)e$. Thus, the kinetic energy is already comparable with the thermal energy. For a cold magnetised flow the speed of magnetic sound (fast magnetosonic wave) is $c_f^2 = B^2/4\pi\rho$ and at the sonic point $\rho v^2 = B^2/4\pi$. Thus, the kinetic energy is already comparable with the magnetic energy.

The relativistic expression for the sound speed is $a_s^2 = (\gamma p/w)c^2$, where $w = \rho c^2 + p\gamma/(\gamma - 1)$ is the gas enthalpy, ρ and p are measured in the fluid frame. The condition of highly-relativistic asymptotic speed requires relativistically hot gas (initially), that is the Lorentz factor of thermal motion $\Gamma_{\rm th} \gg 1$, $p \gg \rho c^2$, and $\gamma \approx 4/3$. This means that at the sonic point $v^2 \approx c^2/3$ and the corresponding Lorentz factor is only $\Gamma^2 \approx 3/2 \ll \Gamma_{\rm th}^2$. Thus, most of the energy is still in the thermal form.

The relativistic expression for the speed of magnetic sound in cold gas is

$$c_f^2 = B'^2/(B'^2 + 4\pi\rho c^2), \tag{1}$$

where B' is the magnetic field as measured in the fluid frame. Thus, at the sonic point $\Gamma^2 = 1 + B'^2/4\pi\rho c^2$. Now one can see that large asymptotic Lorentz factor implies $B'^2/4\pi\rho c^2 \gg 1$ at the sonic point. Indeed, if $B'^2/4\pi\rho c^2 \ll 1$ then not only $\Gamma \approx 1$ but also the magnetic energy per particle is is much less than its rest mass, which means that only a small increase of the Lorentz factor is possible when this magnetic energy is converted into the kinetic one downstream.

Thus, large asymptotic Lorentz factor implies magnetically-dominated flow at the sonic point.

In the subsonic regime different fluid elements can communicate with each other by means of sound waves both along and across the direction of motion. In the supersonic regime the causal connectivity is limited to the interior of the Mach cone and this has an important implication for the efficiency of magnetic acceleration.

2.2. Acceleration and differential collimation

Consider first the thermal acceleration of relativistic flows in the supersonic regime. The mass and energy conservation laws for steady-state flows imply that both the mass and the total energy fluxes are constant along the jet:

$$\rho\Gamma cA = \text{const}, \tag{2}$$

$$(\rho c^2 + 4p)\Gamma^2 cA = \text{const}, \tag{3}$$

where $A \propto R_j^2$ is the jet cross section area, R_j is the jet radius, jet speed $v \approx c$, and for simplicity we assume $\gamma = 4/3$. From these equations it follows that

$$(1 + 4p/\rho c^2)\Gamma = \Gamma_{\max} \tag{4}$$

(this is known as the Bernoulli equation). $\Gamma_{\max}$ is the constant that equals to the Lorentz factor of the flow after complete conversion of its thermal energy into the kinetic one. From the mass conservation we find that $\rho \propto \Gamma^{-1}R_j^{-2}$ and thus

$$p/\rho \propto \rho^{1/3} \propto \Gamma^{-1/3}R_j^{-2/3}. \tag{5}$$

When $p \gg \rho$, allowing plenty of thermal energy to be spent on continued plasma acceleration, this equation and the Bernoulli equation yield

$$\Gamma \propto R_j \tag{6}$$

Thus, the sideways (or transverse) jet expansion is followed by rapid acceleration. In particular, for freely expanding conical jets $\Gamma \propto z$ there z is the distance along the jet.

For cold magnetised flows the energy equation can be written as

$$(\rho c^2 + B'^2/4\pi)\Gamma^2 cA = \text{const}, \tag{7}$$

where we ignore the contribution due to the small poloidal component of the magnetic field[1]. The Bernoulli equation then reads

$$(1 + B'^2/4\pi\rho c^2)\Gamma = \Gamma_{\max}, \tag{8}$$

where $\Gamma_{\max}$ is the Lorentz factor after complete conversion of the magnetic energy into the kinetic one. Assuming that the radius of all streamlines evolves like R_j and using the magnetic flux freezing condition one finds the familiar law for the evolution of transverse magnetic field $B \propto R_j^{-1}$ and $B' = B/\Gamma \propto \Gamma^{-1}R_j^{-1}$. This gives us $B'^2/\rho = R_j^0\Gamma^{-1}$ and then the Bernoulli equation yields the uncomfortable result

$$\Gamma = \text{const}. \tag{9}$$

The same conclusion can be reached in a slightly different way. When a fluid element expands only sideways, its volume grows as $V \propto R_j^2$ and its magnetic energy $e_m \propto B^2V \propto R_j^0$ remains unchanged, implying no conversion of the magnetic energy and no acceleration. Thus, in contrast to the thermal case, the sideways expansion of a cold magnetised flow is not sufficient for its acceleration.

For the magnetic mechanism to work a special condition, which can be described as *differential collimation*, has to be satisfied. In order to see this we refine our analysis and consider the flow between two axisymmetric flow surfaces with cylindrical radii $r(z)$ and $r(z) + \delta r(z)$ (Fig.1).
Now $A \propto r\delta r$, $\rho \propto \Gamma^{-1}(r\delta r)^{-1}$, $B \propto \delta r^{-1}$, $B'^2/\rho \propto (r/\delta r)\,\Gamma^{-1}$ and the magnetic Bernoulli equation yields

$$\Gamma \simeq \Gamma_{\max}\left(1 - \frac{r}{\delta r}\frac{\delta r_0}{r_0}\right), \tag{10}$$

where r_0 and δr_0 are the initial surface parameters and we assume that the initial Lorentz

[1] This is sufficiently accurate when $R_j \gg R_{\rm LC}$, the light cylinder radius, the condition which is normally satisfied in the supersonic regime.

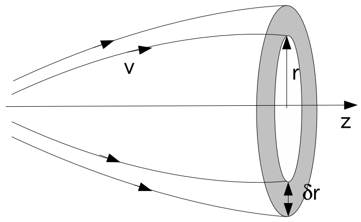

Fig. 1. Flow surfaces of steady-state axisymmetric jet.

factor $\Gamma_0 \ll \Gamma_{\rm max}$. This result shows that the magnetic acceleration requires $r/\delta r$ to decrease with the distance along the jet. In other words, the separation between neighbouring flow surfaces should increase faster than their radius. For example, consider a flow with parabolic flow surfaces, $z = z_0(r/r_0)^a$, where the power index varies from surface to surface, $a = a(r_0)$. Then

$$\frac{r}{\delta r} = \frac{r_0}{\delta r_0}\left[1 - \frac{da}{dr_0}\frac{r_0}{a^2}\ln\frac{z}{z_0}\right]^{-1}. \qquad (11)$$

Thus, if $da/dr_0 = 0$, and hence all flow surfaces are "uniformly" collimated, then $r/\delta r$ =const and the magnetic acceleration fails. This includes the important case of ballistic conical flow with radial streamlines. If, however, $da/dr_0 < 0$, and thus the inner flow surfaces are collimated faster compared to the outer ones, then $r/\delta r$ decreases along the jet and the flow accelerates.

Whether such differential collimation can arise naturally depends on the details of the force balance across the jet. Such balance is described by the Grad-Shafranov equation (e.g. Beskin 2009), which is notoriously difficult to solve. Only very few analytic or semi-analytic solutions for rather simple cases have been found so far (e.g. Beskin et al. 1998; Beskin & Nokhrina 2006; Vlahakis & Königl 2003). Recently, this issue has been studied using time-dependent numerical simulations (Komissarov et al. 2007,2009a,b; Tchekhovskoy et al. 2009a,b). Their results shows that the differential collimation can develop under the self-collimating action of magnetic hoop stress associated with the azimuthal magnetic field, but only in cases where efficient externally imposed confinement helps to keep the jet sufficiently narrow. For example, the asymptotic Lorentz factor of conical jets decreases with the opening angle and unless the angle is small enough the jet remains Poynting-dominated. In the case of parabolic jets, the opening angle decreases with distance and the acceleration continues until the kinetic energy becomes comparable with the magnetic one.

When the external confinement is provided by the gas with the pressure distribution $p_{\rm ext} \propto z^{-\alpha}$, where $\alpha < 2$, the jet shape is indeed parabolic $R_j \propto z^{\alpha/4}$ and its Lorentz factor grows as $\Gamma \propto R_j \propto z^{\alpha/4}$ until the energy equipartition is reached (Komissarov et al. 2009a; Lyubarsky 2009). As far as the dependence on R_j is concerned, this is as fast as in the thermal mechanism. However, as a function of z the Lorentz factor grows slower than in the case of thermally-accelerated conical jet. For $\alpha > 2$ the external confinement is insufficient – the jets eventually develop conical streamlines and do not accelerate efficiently afterwards. Various components of the Lorentz force, the hoop stress, magnetic pressure, and electric force, finely balance each other. This is in contrast to the thermal acceleration, which remains efficient for jets with conical geometry.

Although the detailed analysis of this issue is rather involved, one can get a good grasp of it via the causality argument. Indeed, the favourable differential self-collimation can only be arranged if flow surfaces "know" what other flow surfaces do. This information is propagated by fast magnetosonic waves[2]. In subsonic regime, these waves can propagate in all directions and have no problem in establishing causal communication across the jet. In the supersonic regime they are confined to the Mach cone which points in the direction of motion. When the characteristic opening angle of the Mach cone,

$$\sin\theta_{\rm M} = \Gamma_f c_f/\Gamma v, \qquad (12)$$

[2] Alfvén and slow magnetosonic waves transport information only along the magnetic field lines.

where c_f and Γ_f are the fast magnetosonic speed and the corresponding Lorentz factor respectively, becomes smaller than the jet opening angle, θ_j, the communication across the jet is disrupted. Thus, the condition for effective magnetic acceleration is $\theta_j < \theta_{\rm M}$. Using equations 12,1,8, one can write this condition as

$$\Gamma < (\Gamma_{\rm max}/\sin^2\theta)^{1/3}. \tag{13}$$

For spherical wind with $\Gamma_{\rm max} \gg 1$ this condition reads $\Gamma < \Gamma_{\rm max}^{1/3} \ll \Gamma_{\rm max}$. Thus, the magnetic acceleration of relativistic winds is highly inefficient. Higher efficiency can be reached for collimated flows. For $\Gamma \geq 0.5\Gamma_{\rm max}$ this condition requires

$$\theta_j\Gamma \leq 1, \tag{14}$$

where we used the small angle approximation. For GRB jets, with their deduced Lorentz factors as high as $\Gamma = 10^3$, this leads to $\theta_j \leq 0.06^o$. This is much less than the model-dependent estimates of $2^\circ - 30^\circ$ ($\Gamma\theta_j \sim 7 - 70$), based on the pre-Swift observations of afterglows (Frail, Waxman & Kulkarni 2000; Panaitescu & Kumar 2001). Moreover, the observed ratio of GRB and core-collapse supernova events is $\sim 10^{-5}$ (Woosley & Bloom 2006) and for beaming angles as small as 0.06^o we essentially require that every core-collapse supernova produces GRB. None of the current models of GRB central engines predicts such a high rate of GRBs and radio surveys of local SNe Ibc show that no more than 3% of them harbour relativistic ejecta (Berger 2003). Thus, either GRB jets remain magnetically-dominated all the way, or other acceleration mechanisms come into play. During the transition from confined to unconfined state, which may occur when a GRB jet crosses the surface of collapsing star, a rarefaction wave moves into the jet. It produces favourable differential collimation, due to the fact that the outer flow surfaces straighten up earlier compared to the inner ones, and the flow experiences additional acceleration (Komissarov et al. 2009b; Tchekhovskoy et al. 2009b). Based on the results of numerical simulations, one can expect an increase of $\Gamma\theta_j$ at most by a factor of ten, largely via increase of Γ. This shifts the theoretical value of θ_j closer to the observed range, but it is still a bit low. Moreover, asymptotically the flow remains rather highly magnetised, with approximate equipartition between the kinetic and magnetic energy at best. This makes shock dissipation rather ineffective.

Condition 14 is satisfied by AGN jets, where $<\theta_j\Gamma> \simeq 0.26$, with significant spread around this value (Pushkarev et al. 2001). However, the observed linear polarization angles (EVPA) of AGN jets seem to present another problem for the standard model. In approximately half of all cases, the electric field vector is normal to the jet direction (Wardle 1998). This means that in the comoving jet frame the longitudinal component of magnetic field is at least comparable to the transverse one (Lyutikov et al. 2005). On the other hand, the standard model predicts that beyond the light cylinder (LC), $B_\phi/B_p \simeq (R_j/R_{\rm LC})$, where B_p is the poloidal (predominantly longitudinal) and B_ϕ is the azimuthal components of magnetic field as measured in the observer's frame. In the comoving jet frame this leads to $B'_\phi/B'_p \simeq \Gamma^{-1}R_j/R_{\rm LC}$. For a rapidly rotating black hole $R_{\rm LC} \simeq 4R_g$, where $R_g = GM/c^2$ is the hole's gravitational radius (e.g. Komissarov 2004). This leads to

$$\frac{B'_\phi}{B'_p} \simeq \frac{10^3}{\Gamma}\left(\frac{\theta_j}{1^\circ}\right)\left(\frac{l_j}{1{\rm pc}}\right)\left(\frac{M}{10^8 M_\odot}\right)^{-1}, \tag{15}$$

where l_j is the distance from the black hole. Thus, unless the AGN jets are produced by very slowly rotating black hole holes, which is highly unlikely, the standard model is in conflict with observations.

According to the numerical simulations the asymptotic magnetisation of jets with $\Gamma_{\rm max} \sim 20$, typical for AGNs, is somewhat lower compared to that of jets with $\Gamma_{\rm max} \sim 1000$, typical for GRBs. However, the efficiency of shock dissipation is still reduced.

3. Alternatives to the standard model

In addition to the standard model other ideas have been put forward, each relaxing some of the model assumptions and exploring the consequences. Heinz & Begelman (2000) assumed that current-driven instabilities randomise the magnetic field, transferring energy from the slowly decaying transverse component, $B'_\perp \propto \Gamma^{-1}R_j^{-1}$, to the rapidly decaying longitudinal component, $B'_\parallel \propto R_j^{-2}$. As the result, the magnetic field strength evolves as $B' \propto R_j^{-2}$, and $B'^2/\rho \propto R_j^{-2}\Gamma$. Then in the magnetically dominated regime the Bernoulli equation (Eq.8) yields $\Gamma \propto R_j$. Thus, randomised magnetic field behaves as ultrarelativistic gas with $\gamma = 4/3$, providing as rapid magnetic acceleration as the thermal mechanism.

Such instabilities are likely to be followed by magnetic dissipation and plasma heating. This may also facilitate bulk acceleration of jets (Drenkhahn 2002; Drenkhahn & Spruit 2002). In particular, heat is easily converted into kinetic energy during sideways expansion of the jet.

Contopoulos (1995) argued that the magnetic mechanism can be more efficient if the jet is produced not in a steady-state but in an impulsive fashion. He dubbed the impulsive magnetic mechanism "astrophysical plasma gun". Recently, this idea have been explored in the relativistic regime and the results look very promising (Granot et al. 2010; Lyutikov & Lister 2010; Lyutikov 2010). The main features of the "relativistic plasma gun" mechanism are nicely demonstrated in the following simple problem of one-dimensional expansion of highly magnetised plasma into vacuum.

Consider a planar uniform plasma shell of width l_0 with initial magnetization $\sigma_0 = B_0^2/4\pi\rho_0 c^2$, where B_0 is the initial magnetic field and ρ_0 is the initial rest mass density. On the left of the shell is a solid conducting wall and on the right is vacuum, the magnetic field being parallel to the wall. When $\sigma_0 \gg 1$ the particle inertia is very small (we assume that the plasma is cold) and has only a little effect on the evolution of the electromagnetic field, which closely follows the solution of vacuum electrodynamics. In the vacuum solution, an electromagnetic pulse of width $l_1 = 2l_0$ with constant magnetic and electric field $B = E = B_0/2$ separates from the wall at time $t_0 \simeq l_0/c$. The corresponding exact relativistic MHD solution for $\sigma_0 = 30$ at this time is presented in figure 2. One can see that indeed $B \approx B_0/2$. However, close to the right front of the pulse a significant fraction of magnetic energy is already converted into the kinetic energy of the plasma. In fact, the magnetization parameter $\sigma \to 0$ at the front and $\Gamma \to 1 + 2\sigma_0$. The plasma acceleration is driven by the gradient of magnetic pressure (Although, in the laboratory frame the magnetic field is almost uniform the magnetic pressure is given by the strength of magnetic field in the comoving frame, B', which is non-uniform.).

After the separation, a secondary rarefaction wave begins to move inside the pulse from the left (In Fig.3 , which shows the solution at $t = 20t_0$, its front is located at $x = 18$.) and the pulse sheds plasma into the low density tail. However, this process is very slow and to first approximation the pulse rest mass, as well as its total energy and momentum, which are mainly in the electromagnetic form, are constant. However, the shell plasma continues to be accelerated by the magnetic pressure gradient that has developed before the separation. The rate at which the electromagnetic energy-momentum is transferred to plasma is dictated by the rate of the longitudinal expansion of the shell, which is given by

$$\frac{dl}{dt} = v_h - v \simeq \frac{c}{2\Gamma^2} , \tag{16}$$

where v_h is the constant speed of the vacuum interface and v is the characteristic (mean) speed of the shell plasma, and in the approximation we assume that $\Gamma_h \gg \Gamma \gg 1$. The electromagnetic energy of the shell, $\mathcal{E}_m \simeq lB^2/4\pi \simeq \mathcal{E}_{m,0}(l_1/l)$, and its kinetic energy $\mathcal{E}_k = \mathcal{E}_{m,0} - \mathcal{E}_m \simeq \mathcal{E}_{m,0}(1 - (1 + X)^{-1})$, where $X = (l - l_1)/l_1$ and $\mathcal{E}_{m,0}$ is the initial magnetic energy. As long as the electromagnetic energy dominates we have $X \ll 1$ and can use the approximation $\mathcal{E}_k \simeq \mathcal{E}_{m,0}X$. On the other hand $\mathcal{E}_k \simeq Mc^2\Gamma$, where $M = \rho\Gamma l$ is the shell rest mass. Since the secondary rarefaction, which develops at the back of the shell, crosses the shell very slowly, one may assume that M is constant. Combining the last two equations we find that $X \simeq M\Gamma/\mathcal{E}_{m,0}$ and this allows us to write Eq.16 as

$$\Gamma^2 \frac{d\Gamma}{dt} = a , \tag{17}$$

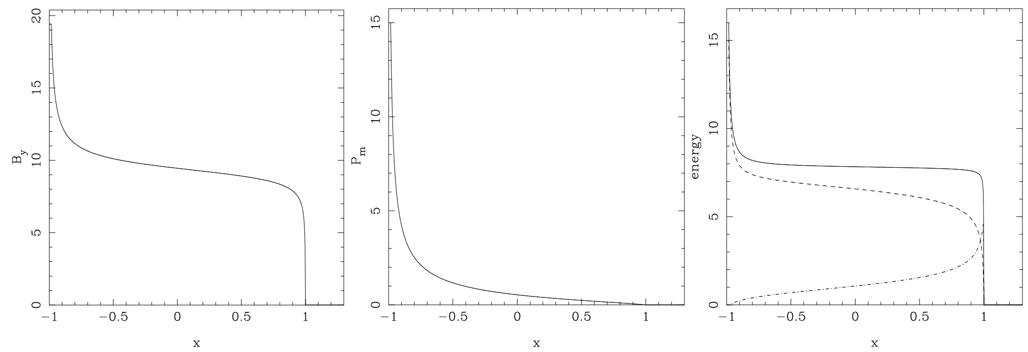

Fig. 2. Solution at the time of separation $t = t_0$. The units are such that the dimensionless parameters $l_0 = 1$, $\rho_0 = 1$ and $c = 1$. The wall is located at $x = -1$ and the initial vacuum interface is at $x = 0$. The top row shows (from left to right) the magnetic field B_y, the Lorentz factor Γ, and the local magnetization parameter, $\sigma = (B')^2/4\pi\rho$, where B' is the magnetic field in the fluid frame. The bottom row shows (from left to right) the flow velocity v_x, magnetic pressure, and the densities of total energy (*solid line*), magnetic energy (*dashed line*), and kinetic energy (*dash-dotted line*) as measured in the wall frame.

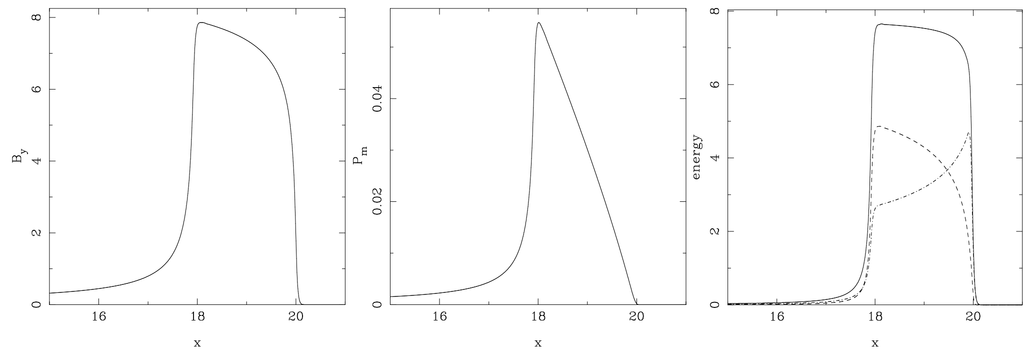

Fig. 3. The same as in Fig. 2 but at $t = 20t_0$.

where $a = \mathcal{E}_{m,0}c/2Ml_1 = \sigma_0/8t_0$. Integrating this equation we find that for $t \gg t_0$

$$\Gamma \simeq \sigma_0^{1/3}\left(\frac{t}{t_0}\right)^{1/3}, \tag{18}$$

where we ignored the factor of order unity, which is justified given the approximate nature of our calculations. The corresponding evolution of the shell thickness

$$l \sim l_1\left(1 + \sigma_0^{-2/3}\left(\frac{t}{t_0}\right)^{1/3}\right). \tag{19}$$

(This is what the shell thickness would be as the result of the expansion of plasma inside the shell. The secondary rarefaction reduces the shell thickness below this estimate.) The condition $X \ll 1$ is no longer satisfied when t exceeds $t_c \equiv \sigma_0^2 t_0$ as for $t = t_c$ Eq.19 gives $l = 2l_1$. At this point the shell evolution changes. Formal application of Eq.18 gives the Lorentz factor $\Gamma = \sigma_0$ at $t = t_c$. This is larger than the value corresponding to full conversion of electromagnetic energy, $\Gamma_c \simeq \mathcal{E}_{m,0}/Mc^2 = \sigma_0/2$. This shows that around time t_c the growth of Lorentz factor begins to saturate and the shell enters the coasting phase. Since Γ_c is still significantly lower compared to the Lorentz factor of the leading front of the shell, $\Gamma_h \sim 2\sigma_0$, the shell thickness is now governed by the equation

$$\frac{dl}{dt} = \frac{c}{2\Gamma_c^2} \,. \tag{20}$$

Integrating this equation and applying the initial condition $l(t_c) = 2l_1$ we find

$$l = l_1 \left(2 + \frac{t - t_c}{t_c}\right), \ \mathcal{E}_m = \mathcal{E}_{m,0} \left(2 + \frac{t - t_c}{t_c}\right)^{-1} .$$

For $t \gg t_c$ these equations give

$$\frac{l}{l_0} \simeq 2\left(\frac{t}{t_c}\right), \ \frac{\mathcal{E}_m}{\mathcal{E}_{m,0}} \simeq \left(\frac{t}{t_c}\right)^{-1}, \ \sigma \simeq \left(\frac{t}{t_c}\right)^{-1} . \tag{21}$$

Thus, at $t \sim 10t_c$ essentially all electromagnetic energy is converted into the kinetic energy of plasma. If the jet production is indeed highly intermitted and the separation between different shells is significantly larger than their thickness then the shock dissipation during the coasting regime can be very effective. Not only the flow magnetization becomes very low but the variation of the Lorentz factor $\Delta\Gamma \sim \Gamma$, allowing dissipation of a significant fraction of kinetic energy.

4. Discussion and conclusions

It appears that the properties and the potential of the magnetic mechanism within the framework of the standard model, which deals with steady-state axisymmetric ideal MHD flows, is now well understood. This mechanism is not as fast and robust as the thermal mechanism. In order to be efficient, it requires external confinement, which has to ensure that the jet remains sufficiently narrow to be causally connected in the transverse direction. When the external pressure distribution is a power law, $p_{\text{ext}} \propto z^{-\alpha}$, the power index has to be below $\alpha_c = 2$. Conical jets, which arise when such confining medium is not present, is an example where the standard model may fail.

The fact that the standard model has been in the focus of theoretical studies for so many years is merely a reflection of its relative simplicity. Such issues as the jet stability, variability of central engine, and inhomogeneity of external medium have always been in the back mind of researches but it was assumed that these are details that can be considered later on, when the key issue of acceleration would have been settled. Now it appears that the standard model could be an oversimplification. The fine balance of forces in this model, which leads to the reduced efficiency of magnetic acceleration, may not be representative of the magnetic mechanism in general. Instead, it may be specific to the standard model, reflecting its strict symmetries. As we have seen, both randomization of magnetic field, which is a natural outcome of magnetic instabilities, and impulsive operation of the jet engine, are actually capable of increasing effectiveness and robustness of the magnetic mechanism. Moreover, the observations of both AGN and GRB jets seem to require a less restrictive model, both in terms of polarization and jet opening angle.

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