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Alfvén wave amplification and self-containment of cosmic-rays escaping from a supernova remnant

Y. Fujita[1], F. Takahara[1], Y. Ohira[2], and K. Iwasaki[3]

[1] Department of Earth and Space Science, Graduate School of Science, Osaka University, 1-1 Machikaneyama-cho, Toyonaka, Osaka 560-0043, Japan
e-mail: fujita@vega.ess.sci.osaka-u.ac.jp
[2] Theory Centre, Institute of Particle and Nuclear Studies, KEK, 1-1 Oho, Tsukuba 305-0801, Japan
[3] Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Abstract. We investigate the escape of cosmic-rays (CRs) accelerated at the shock front of a supernova remnant (SNR). For that purpose, we solve a transport equation for the CRs from the vicinity of the shock front to the region far away from the front. We consider the amplification of Alfvén waves through CR streaming and the scatter of CRs by the waves. We found that the waves grow enough to delay the propagation of the CRs even far away from the shock front. Our results indicate that the γ-ray spectra observed at molecular clouds illuminated by the CRs escaped from SNRs are significantly affected by the wave amplification.

Key words. ISM: clouds – cosmic rays – ISM: supernova remnants

1. Introduction

It is often considered that cosmic-rays (CRs) in the Galaxy are accelerated at supernova remnants (SNRs). Recently, γ-rays have been detected from SNRs. For example, TeV γ-rays have been observed with H.E.S.S around SNRs (Aharonian et al. 2004, 2005). GeV γ-rays have also been observed with Fermi and AGILE (Abdo et al. 2009, 2010; Giuliani et al. 2010).

In Fujita et al. (2009), we studied γ-ray spectra of molecular clouds around SNR W 28 and another possible SNR hidden in the open cluster Westerlund 2. The γ-rays seem to be emitted from CRs escaped from the SNRs. In this study, we compared the timescale of the evolution of the SNRs with that of CR diffusion around them. Here, we assume that the diffusion coefficient is

$$\kappa_{\rm ISM}(E) = 10^{28}\chi\left(\frac{E}{10\,{\rm GeV}}\right)^{0.5} \times\left(\frac{B}{3\,\mu{\rm G}}\right)^{-0.5} {\rm cm^2\,s^{-1}}\,, \quad (1)$$

where E is the CR energy, and B is the magnetic field (Gabici, Aharonian & Casanova 2009). For the typical region in the Milky-way, we expect that $\chi \sim 1$. If we assume $\chi \sim 1$, diffusion time of CRs for a cloud with the size

Send offprint requests to: Y. Fujita

of $\sim$ 15 pc is $\sim$ 100 yr. On the other hand, active particle acceleration appears to have ended $\sim 10^4$ yr ago for these two SNRs. Since the clouds around the SNRs are still γ-ray bright, we speculated that $\chi \sim 0.01$ and thus the diffusion time is extended to $\sim 10^4$ yr.

Motived by these estimations, we constructed a simple model (Fujita et al. 2010). In this model, we assume that Alfvén waves excited through CR streaming scatter CRs and reduce the diffusion coefficient. We were interested in the escape of CRs into interstellar space or regions far away from the shock front. We treated the propagation of CRs accelerated at the shock front and amplification of Alfvén wave at the same time. The results indicated that CRs cannot leave the SNR for $\sim 10^4$ yr.

However, the model of Fujita et al. (2010) is rather simple. In particular, we separately treated the acceleration of CRs and their escape into interstellar space. Since both of the processes follow the same transport equation, they should be treated seamlessly. Therefore, in this study, we solve the transport equation from the vicinity of the shock front to the region far away from the front. Moreover, we consider the Alfvén wave amplification as we did in our previous studies. The details of this study are shown in Fujita et al. (2011).

2. Models

We consider only protons as CRs and neglect electrons. The transport equation for CRs is

$$\frac{\partial f}{\partial t} = \nabla(\kappa\nabla f) - \boldsymbol{w}\nabla f + \frac{\nabla \boldsymbol{w}}{3} p \frac{\partial f}{\partial p} + Q \,, \qquad (2)$$

where $f(r, p, t)$ is the distribution function of the particles, p is the momentum, κ is the diffusion coefficient, $\boldsymbol{w}$ is the hydrodynamic velocity of the background gas, and $Q = Q_0\delta(r-R_s)$ is the source term for the particles, which are injected at the shock front ($r = R_s$) at a momentum of $p_{\rm inj}$. The coefficient Q_0 is given by

$$Q_0 = \epsilon \frac{\rho_1 u_1}{m} \frac{\delta(p - p_{\rm inj})}{4\pi p_{\rm inj}^2} \,, \qquad (3)$$

where ρ_1 and u_1 are the gas density and the gas velocity relative to the shock just upstream the shock front, respectively. We assume that $p_{\rm inj} = 2\, m c_{s2}$, where c_{s2} is the sound velocity behind the shock front. The fraction of gas particles that go into the acceleration process at the shock front is $\epsilon = 10^{-4}$.

The equations for the background gas are

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \boldsymbol{w}) = 0 \,, \qquad (4)$$

$$\rho \frac{\partial \boldsymbol{w}}{\partial t} + \rho(\boldsymbol{w} \cdot \nabla)\boldsymbol{w} = -\nabla(P_c + P_g) \,, \qquad (5)$$

$$\frac{\partial P_g}{\partial t} + (\boldsymbol{w} \cdot \nabla)P_g + \gamma_g(\nabla \boldsymbol{w})P_g = 0 \,, \qquad (6)$$

where ρ, γ_g, and P_g are the density, specific heat capacity ratio, and pressure of the gas. The CR pressure is given by

$$P_c = \frac{4\pi c}{3} \int_{p_{\rm min}}^{\infty} dp \frac{p^4 f}{\sqrt{p^2 + m^2c^2}} \,, \qquad (7)$$

where $p_{\rm min}$ is the minimum momentum of injected CR particles, m is the mass of the particles, and c is the velocity of light. We solve these equations based on the numerical method developed by Berezhko, Yelshin, & Ksenofontov (1994).

In our model, we do not fix the diffusion coefficient κ. As Alfvén waves are amplified by the CR streaming instability, the coefficient can be affected by the amplification, because waves scatter CR particles.

The growth of the waves is given by

$$\frac{\partial \psi}{\partial t} \approx \frac{4\pi}{3} \frac{v_{\rm A} p^4 v}{U_{\rm M}} |\nabla f| \,, \qquad (8)$$

where $\psi(t, \boldsymbol{r}, p)$ is the energy density of Alfvén waves per unit logarithmic bandwidth (which are resonant with particles of momentum p) relative to the ambient magnetic energy density $U_{\rm M}$, and $v_{\rm A}$ is the Alfvén velocity (Skilling 1975; Bell 1978). We do not consider the damping of the waves for simplicity. The diffusion coefficient is

$$\kappa = \frac{4}{3\pi} \frac{pvc}{eB_0\psi} \frac{\rho_0}{\rho} \,, \qquad (9)$$

where v is the velocity of the particle, e is the elementary charge, B_0 is the unperturbed magnetic field, and ρ_0 is the density of unperturbed ISM.

We do not start our calculations with $\psi = 0$, because waves do not grow enough for CR acceleration. Thus, for the region far away from the shock front, we adopt $\psi = \psi_{\rm ISM}$ as the initial value of ψ, where $\psi_{\rm ISM}$ gives the diffusion coefficient for the typical region in the Galaxy ($\chi \sim 1$ in equation [1]). For the vicinity of the shock front, we found that the diffusion coefficient must be close to that for the Bohm diffusion from the beginning of the calculations. If the diffusion coefficient is much smaller than that, CRs cannot be accelerated to high energies. Therefore, we assume the initial condition of $\psi = \psi_{\rm B}$ at the shock front, where $\psi_{\rm B}$ is the wave density corresponding to the Bohm diffusion. For the region between the shock front and the interstellar space, we interpolate the above wave densities ($\psi_{\rm B}$ and $\psi_{\rm ISM}$, see equation 11 of Fujita et al. 2011). We assumed that the diffusion is the Bohm one for $r - R_s \lesssim a_i \kappa_{\rm B0}/V_s$, where $\kappa_{\rm B0}$ is the Bohm diffusion coefficient, and V_s is the shock velocity. Although the value of a_i cannot be specified, it would not be much larger than 1. Therefore we assume $a_i = 5$ in the following simulations.

We consider three models: (A) the growth of ψ is considered, (B) the diffusion coefficient is fixed at the Bohm values regardless of time and position, and (C) the wave energy density does not change and is fixed at the initial values. The models B and C are calculated for comparison.

The density and sound velocity of the background ISM is $\rho_0 = 7.0 \times 10^{-27}$ g cm^{-3} and $c_s = 154$ km s^{-1}, respectively. Since the temperature is relatively high, we do not consider the neutral damping of the waves. A supernova explodes at $t = 0$ and $r = 0$ with an energy of 10^{51} erg. The background magnetic field is $B_0 = \sqrt{8\pi U_M} = 3\,\mu$G.

3. Results

Fig. 1 shows the profiles of wave energy density ψ at $t = 10\,t_0$, where t_0 is the time when the free expansion phase of the SNR ends. At $r = R_s$, the density is $\psi \approx 4/\pi$, which corresponds to the Bohm diffusion (Fujita et al. 2011). At the right ends of the curves, $\psi = \psi_{\rm ISM}$. The deference between Model A and C

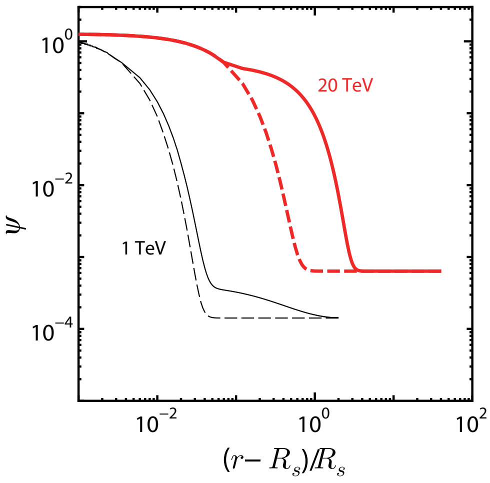

Fig. 1. Profiles of wave energy density ψ at $t = 10\,t_0$. Thin lines correspond to waves interacting with particles with $pc = 1$ TeV and thick lines correspond to those with $pc = 20$ TeV. Solid lines are for Model A and dashed lines are for Model C (Fujita et al. 2011).

indicates that waves are significantly amplified. In particular, waves that interact with particles with $pc = 20$ TeV are amplified even at $r \sim 2\,R_s$. However, the way they grow is somewhat different between 1 TeV and 20 TeV (Fig. 1). For the former, they amplify at the 'instep' where gradient of ψ is small ($0.02 \lesssim (r - R_s)/R_s \lesssim 2$), while for the latter, they amplify at the 'shin' where gradient of ψ is large ($0.1 \lesssim (r - R_s)/R_s \lesssim 0.6$).

The difference is made by the energy dependence of $\psi_{\rm ISM}$. From equations (1) and (9), one can obtain $\psi_{\rm ISM} \propto E/\kappa_{\rm ISM}(E) \propto E^{0.5}$. This means that $\psi_{\rm ISM}$ is an increasing function of E. Because of this, lower energy particles can go farther away from the shock front, if the distance is represented in the units of the ratio of the Bohm diffusion coefficient to the shock velocity (Fujita et al. 2011). The particle distribution shown in Fig. 2 reflects this. Lower energy particles ($pc = 1$ TeV) can escape into the instep region, where $\psi \sim \psi_{\rm ISM}$, while higher energy particles ($pc = 20$ TeV) cannot and confined in the shin region. Thus, ψ increases in the instep for $pc = 1$ TeV and it increases in the shin for $pc = 20$ TeV (Fig. 1). Fig. 3 shows

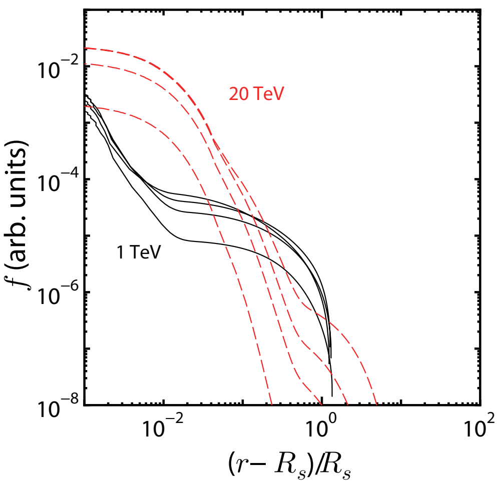

Fig. 2. Profiles of CR distribution f at $t = 0.60\,t_0$, $0.65t_0$, $0.70t_0$, and $1.0t_0$ (from bottom to top) (Fujita et al. 2011).

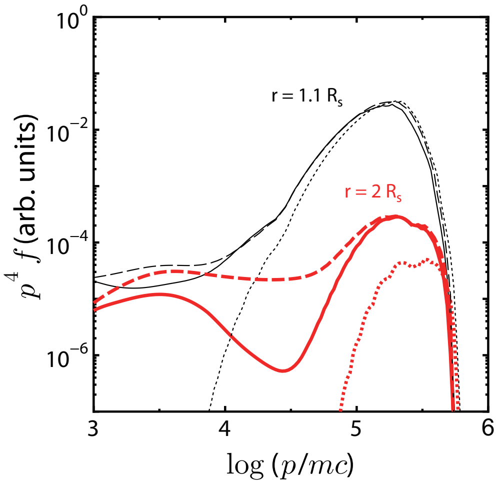

Fig. 3. Spectra of CRs at $r = 1.1\,R_s$ and $2\,R_s$ at $t = 10\,t_0$. Solid, dotted, and dashed lines are for Models A, B, and C, respectively (Fujita et al. 2011).

the energy spectra of CRs at $r = 1.1\,R_s$ and $2\,R_s$. Particles with $\log(p/mc) \sim 5.2$ make a peak, because diffusion for those particles is very fast even if the Bohm diffusion is assumed (Model B). For Models A and C, lower energy particles can also reach those radii because the diffusion coefficient is larger than that for the Bohm diffusion. However, the growth of waves makes the diffusion slower for Model A compared to Model C, which is clearly seen at $\log(p/mc) \sim 4.4$ and $r = 2\,R_s$.

4. Conclusions

We studied the escape of CRs from a SNR. We consider scattering of CRs by Alfvén waves amplified by the CRs. We solved a transport equation for CRs from the vicinity of the shock front to the region far away from the front. We found that waves are amplified even at $r \sim 2R_s$, which makes the CR diffusion slower. We suppose that this delay of escape influences the CRs that are responsible for the γ-ray emission observed around SNRs.

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