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Equipartition calculation for supernova remnants

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Abstract. Equipartition or the minimum-energy calculation is a widespread method for estimating magnetic field strength and energy in the magnetic field and cosmic rays particles by using only the radio synchrotron emission. Despite of its approximate character, it remains a useful tool in situations when no other data about the source are available. In this paper we give a modified calculation which we think is more appropriate for estimating magnetic fields and energetics of supernova remnants.

Key words. ISM: magnetic fields - supernova remnants - radio continuum: general

1. Introduction

Details of equipartition and revised equipartition calculation for radio sources in general are available in Pacholczyk (1970) and Beck & Krause (2005), respectively. A discussion on whether equipartition of energy is fulfilled in real sources and how reliable magnetic field estimates from equipartition calculation are can be found in Duric (1990). We are relying on Bell's theory of diffusive shock acceleration (Bell 1978) and his assumption concerning injection of particles into the acceleration process to derive a slightly modified equipartition i.e. minimum-energy calculation applicable to 'mature' supernova remnants ($v_s \ll 6000-7000$ km/s) with spectral index $0.5 < \alpha < 1$ (energy index $2 < \gamma < 3$).

2. Analysis and results

Following Bell (1978) we will assume that a certain number of particles have been injected into the acceleration process all with the same injection energy $E_{inj} \approx 4\frac{1}{2}m_p v_s^2$.[1] If we assume that shock velocity is low enough so that $E_{\rm inj} \ll m_e c^2$ (and $p^e_{\rm inj} \ll m_e c$), for energy density of a cosmic rays species (e.g. electrons, protons, heavier ions), assuming power-low momentum distribution, we have

$$\begin{aligned}\epsilon &= \int_{p_{\rm inj}}^{p_\infty} 4\pi k p^{-\gamma}(\sqrt{p^2c^2+m^2c^4}-mc^2)\mathrm{d}p \\ &\approx \int_0^\infty 4\pi k p^{-\gamma}(\sqrt{p^2c^2+m^2c^4}-mc^2)\mathrm{d}p \\ &= K(mc^2)^{2-\gamma}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}. \qquad (1)\end{aligned}$$

where K is constant in power-low energy distribution $N(E) = KE^{-\gamma}$. Total cosmic rays energy density is then

$$\epsilon_{\rm CR} = K_e(m_e c^2)^{2-\gamma}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}(1+\kappa), \qquad (2)$$

[1] We assume fully ionized, globally electro-neutral plasma.

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$$\kappa = \left(\frac{m_p}{m_e}\right)^{(3-\gamma)/2} \frac{\sum_i A_i^{(3-\gamma)/2}\nu_i}{\sum_i Z_i \nu_i}, \tag{3}$$

ν_i are ion abundances, A_i and Z_i are mass and charge numbers of elements and we assumed that at high energies $K_p/K_e \approx (n_p/n_e)(m_p/m_e)^{(\gamma-1)/2}$. Note that we have neglected energy losses.

Emission coefficient for synchrotron radiation is, on the other hand,

$$\varepsilon_\nu = c_5 K_e (B\sin\Theta)^{(\gamma+1)/2}\left(\frac{\nu}{2c_1}\right)^{(1-\gamma)/2}, \tag{4}$$

where c_1, c_3 and $c_5 = c_3\Gamma(\frac{3\gamma-1}{12})\Gamma(\frac{3\gamma+19}{12})/(\gamma+1)$ are defined in Pacholczyk (1970). We will use the flux density which is defined as

$$S_\nu = \frac{L_\nu}{4\pi d^2} = \frac{\frac{4\pi}{3}R^3 f\mathcal{E}_\nu}{4\pi d^2} = \frac{4\pi}{3}\varepsilon_\nu f\theta^3 d, \tag{5}$$

where f is volume filling factor, $\theta = R/d$ is angular radius and d is the distance.

If we assume isotropic distribution for the orientation of the pitch angles (Longair 1994) we can take for the average $\langle(\sin\Theta)^{(\gamma+1)/2}\rangle$

$$\frac{1}{2}\int_0^\pi (\sin\Theta)^{(\gamma+3)/2} d\Theta = \frac{\sqrt{\pi}}{2}\frac{\Gamma(\frac{\gamma+5}{4})}{\Gamma(\frac{\gamma+7}{4})}. \tag{6}$$

For the total energy we have

$$E = \frac{4\pi}{3}R^3 f(\epsilon_{\mathrm{CR}} + \epsilon_B), \quad \epsilon_B = \frac{1}{8\pi}B^2. \tag{7}$$

Looking for the minimum energy with respect to B gives

$$E_B = \frac{\gamma+1}{4}E_{\mathrm{CR}}, \quad E_{\min} = \frac{\gamma+5}{\gamma+1}E_B, \tag{8}$$

and

$$B = \Bigg(\frac{3}{2\pi}\frac{(\gamma+1)\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{(\gamma-1)\Gamma(\frac{\gamma+5}{4})}\frac{S_\nu}{fd\theta^3}\cdot$$
$$\cdot(m_e c^2)^{2-\gamma}\frac{(2c_1)^{(1-\gamma)/2}}{c_5}(1+\kappa)\nu^{(\gamma-1)/2}\Bigg)^{2/(\gamma+5)}, \tag{9}$$

or

$$B\ [\mathrm{Ga}] \approx \Bigg(6.286\cdot 10^{(9\gamma-79)/2}\cdot \tag{10}$$
$$\cdot\ \frac{\gamma+1}{\gamma-1}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{\Gamma(\frac{\gamma+5}{4})}\cdot$$
$$\cdot\ (m_e c^2)^{2-\gamma}\frac{(2c_1)^{(1-\gamma)/2}}{c_5}(1+\kappa)\cdot$$
$$\cdot\ \frac{S_\nu[\mathrm{Jy}]}{fd[\mathrm{kpc}]\theta[\mathrm{arcmin}]^3}\nu[\mathrm{GHz}]^{(\gamma-1)/2}\Bigg)^{2/(\gamma+5)},$$

where $m_e c^2 \approx 8.187\cdot 10^{-7}$ ergs.

Our approach is similar to Beck & Krause (2005) in a sense that we do not integrate over frequencies as Pacholczyk (1970), however,

(i) we assume power-low spectra $n(p) \propto p^{-\gamma}$ and integrate over momentum to obtain energy densities of particles,
(ii) we take into account different ion species and not just equal number of protons and electrons at injection (e.g. for H to He ratio 10:1 there is more energy in α-particles then in electrons),
(iii) we use flux density at a given frequency and assume isotropic distribution of the pitch angles for the remnant as a whole.

Incorporating the dependence $\epsilon = \epsilon(E_{\mathrm{inj}})$, which would make formula applicable to the younger remnants, we leave for future work.

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References

Beck R., Krause M., 2005, AN, 326, 414
Bell A. R., 1978, MNRAS, 182, 443
Duric N., 1990, IAUS, 140, 235
Longair M. S., 1994, High Energy Astrophysics Vol. 2, Cambridge Univ. Press, Cambridge
Pacholczyk A. G., 1970, Radio Astrophysics, Freeman and Co., San Francisco