Mem. S.A.It. Vol. 82, 830

Memorie della

Turbulence in the interstellar medium

P. Hennebelle

Laboratoire de radioastronomie, UMR 8112 du CNRS,
École normale supérieure et Observatoire de Paris, 24 rue Lhomond,
75231 Paris cedex 05, France, e-mail: patrick.hennebelle@ens.fr

Abstract. We review our knowledge of the interstellar turbulence emphasing mainly theoretical aspects. Starting with a basic description of incompressible hydrodynamical and magnetized turbulence, we describe our current understanding of highly supersonic isothermal turbulence thought to be relevant in the context of molecular clouds and finally consider the more complicated but more realistic multi-phase magnetized flows.

Key words. Hydrodynamics - Instabilities - Interstellar medium: kinematics and dynamics - structure - clouds

1. Introduction

Turbulence is an ubiquitous process in astrophysics. Indeed most astrophysical fluids are thought to be turbulent. As a consequence, turbulent processes have many important implications ranging from cosmic rays to accretion disks, stars and planets. In spite of its broad range of applications, turbulence is still poorly understood. Here we give a short review of our knowledge of the interstellar turbulence. Recent and comprehensive reviews of interstellar turbulence particularly stressing its role on the star formation process can be found in Elmegreen & Scalo (2004), Scalo & Elmegreen (2004), Mac Low & Klessen (2004) and McKee & Ostriker (2007).

The plan is as follows. First we quickly describe some general principles of incompressible turbulence, a paradigm which is of great importance for terrestrial flows. Since interstellar turbulence is both magnetized and highly compressible, we describe specifically some aspects of the magnetized incompressible turbulent flows and of the compressible ones, distinguishing between the isothermal and the multi-phase flows.

2. Incompressible and unmagnetized turbulence

For obvious reasons, incompressible, unmagnetized turbulence has received considerable efforts and attention. It constitutes the reference against which other types of turbulence will usually be compared. Many excellent textbooks have been devoted to this topic (e.g. Monin & Yaglom, 1975).

A fluid of viscosity ν becomes turbulent when the rates of viscous dissipation which is $\propto \nu/L^2$ at the energy injection scale, L, is much larger than the energy transfer rate $\propto V/L$, where V is the velocity dispersion at the scale, L. The ratio of these 2 rates is the Reynolds number $R_e = VL/\nu$. In general, when R_e is larger than 10-100 the system becomes turbulent. As the Reynolds number increases, the flow becomes more chaotic. Typically in astro-

Send offprint requests to: P. Hennebelle physics one expects the flows to have Reynolds number larger or of the order of $10^6 - 10^8$.

Kolmogorov theory (Kolmogorov 1941) provides a scaling law which is true in the statistical sense and provides a relation between the relative velocity v_l of fluid elements and their separation l, namely $v_l \simeq l^{1/3}$. This stems for the following facts. Let ϵ be the energy flux injected at large scale in the flow, $\epsilon \simeq V^3/L$. As long as the dissipation term, $\simeq \nu/l^2$ is small, it is expected that ϵ remains unchanged being transmitted to smaller scales by the non linear coupling between the scales. Thus, $v_l \propto l^{1/3}$. An equivalent description is to express the spectrum $E(k)$ as a function of the wavenumber $k \propto 1/l$. The total energy V_L^2 can also be written as $\int E(k)dk$, where $E(k)$ is thus the square of the Fourier transform of V_l. Thus $E(k)k \propto l^{2/3}$ and $E(k) \propto k^{-5/3}$. This relation is now well observed both in laboratory turbulence and numerical simulations. An important consequence is that most of the energy is contained at large scales.

From the velocity-size relation, one can estimate the dissipation scale. Using this relation, the Reynolds number can be written as $R_e = VL/\nu \propto L^{4/3}/\nu$ leading to $R_e(l) = (l/L)^{4/3} R_e(L)$. The dissipation arises at the scale, l_d, to which the Reynolds number is of the order of 1. Thus, $l_d \simeq LR_e(L)^{-3/4}$. This shows that for a Reynolds number of the order of 10^8, the ratio between the injection and dissipation scales is about 10^6. A straightforward consequence is that numerically, it is impossible with the present computing power to simulate flows having Reynolds number larger than $\simeq 10^3 - 10^4$ without using subgrid modeling.

In order to get a complete description of the flow statistics, one needs to predict the exponent of the structure functions defined as v_l^p. The original Kolmogorov theory which assumes that $v_l^l \propto l^{p/3}$ has turned to be too simplistic and deviations from it have been firmly established. The most successful approach to date has been developed by She & Leveque (1994). Considering a self-similar hierarchy of eddies, they were able to establish a robust prediction for the scaling exponents which appear to be compatible with the measurements performed in laboratory turbulent experiment.

3. Incompressible magnetized turbulence

As already stated above, magnetic field plays an important role in a lot of astrophysical flows. In particular, the diffuse interstellar medium appears to be permeated by a magnetic field whose intensity is of the order of $6\mu G$ (Heiles & Troland 2005). This implies that the magnetic energy is typically larger than the thermal energy by a factor of a few.

While the simple Kolmogorov dimensional scaling relation, described above, has proven to be very robust, MHD flows appear to be much more difficult to understand. Indeed, despite more than 35 years of analytical, numerical and observational investigations, the energy spectrum of MHD turbulence remains a subject of controversy. The first attempt to establish such a spectrum has been done by Iroshnikov (1963) and Kraichnan (1965). Based on the fact that for incompressible MHD turbulence, any function $\mathbf{v} \pm \mathbf{b}(\mathbf{r} \pm \mathbf{V}_a t)$ is a solution of the MHD equations, implying that Alfvén wave packets traveling in the same direction along the magnetic field are not interacting, they infer a power spectrum $E(k) \propto k^{-3/2}$, thus slightly shallower than the Kolmogorov one. An essential assumption of the Iroshnikov-Kraichnan approach is that the eddies are isotropic, i.e. have the same spatial extension in the field-parallel and field-perpendicular directions. However, numerical and observational data accumulated for the last 30 years indicate that in MHD turbulence the energy transfer occurs predominantly in the field perpendicular direction (Biskamp 2003). This raises the question whether the picture proposed by Iroshnikov and Kraichnan is grasping the essential physical mechanisms.

An important progress has been performed by Goldreich & Sridhar (1995) who have developed a theory which takes into account the anisotropy of the eddies in MHD. They suggested that as the energy cascade proceeds to smaller scales, turbulent eddies progressively become elongated along the large-scale field. As a consequence, they found that the energy transfer time is reduced, compared to the Iroshnikov-Kraichnan approach, and identical to the Kolmogorov estimate. This leads them to a scaling for the field-perpendicular energy spectrum, $E(k_\perp) \propto k_\perp^{-5/3}$. More recently, this issue has been investigated further in various analytical and numerical studies (e.g. Cho et al. 2002 ; Boldyrev, 2005 ; Boldyrev 2006 ; Lee et al. 2010). and appears to still be a matter of debate. Indeed, even the question of the universality of the energy spectrum in incompressible MHD turbulence appear to be unsolved.

4. A highly compressible ISM

The interstellar medium is not expected to be incompressible. Indeed, during the galactic cycle, the interstellar medium density varies from 10^{-2} cm^{-3} up to eventually stellar densities which represents a density enhancement of more than 25 orders of magnitude. Of course this is not achieved in a single step. Instead, the fluid particles endergo a series of contraction.

Traditionally, the ISM is divided in roughly four phases. The hot ionized gas (HIM) whose density and temperature are about 10^{-2} cm^{-3} and 10^6 K, the warm neutral medium (WNM) which is roughly hundred times denser and colder than the HIM, the cold neutral medium itself roughly hundred times denser and colder than the WNM. The last phase, is the molecular hydrogen. Its typical density goes from 10^3 to 10^6 cm^{-3} or more while its temperature is about 10-30 K.

Due to their very high temperatures which lead to sound speeds respectively equal to about 100 km s^{-1} and 10 km s^{-1}, the HIM and the WNM are typically subsonic or transsonic. The CNM and the molecular gas have much lower sound speeds of the order of 1 km s^{-1} and 0.2 km s^{-1} respectively. Given the velocity dispersion observed in the molecular gas, the rms Mach number is typically of the order of $4 \times (L/1\mathrm{pc})^{0.5}$, where L is the size of the molecular clouds (Larson 1981). Thus, it appears that for the cold gas, turbulence is strongly supersonic.

4.1. Compressible isothermal turbulence

The most common assumption is to assume that the gas is isothermal. Indeed, this appears to be a reasonable assumption for the dense molecular gas and most of the studies have been assuming isothermality. The first important point to be stressed, is that a new quantity is necessary to characterize a compressible flow, namely the Mach number $\mathcal{M}$. It is worth recalling that in a shock of Mach number, $\mathcal{M}$, the isothermal gas undergoes a density enhancement equal to $\mathcal{M}^2$. For example, this implies that for a molecular cloud of few parsecs in size, density contrasts of the order of 100 are common.

Given the complexity of the problem, it is not surprising that very few results have been rigorously established analytically and most of our understanding comes from numerical simulations, which have been performed during the last two decades. The biggest simulation performed to date is the one by Kritsuk et al. (2007) and has 2048^3 computing cells. In the one dimensional case, one expects the power spectrum of v, the velocity to be $\propto k^{-2}$. The reason is that a Fourier transform of an Heaviside function is proportional to k^{-1}. Thus the energy spectrum is slightly stiffer than the Kolmogorov spectrum. In 3D, high resolution numerical simulations like the ones performed by Kritsuk et al. (2007), one finds that the powerspectrum of v, is typically $\propto k^{-3.8}$ (note that in 3D, the corresponding exponent for incompressible flow is 11/3). Thus, the exponent is bracketed by the values of incompressible Kolmogorov turbulence and the value of the fully compressible Burgers turbulence. This is not very surprising since even supersonic flows tend to have a large energy fraction in the solenoidal or incompressible modes. A very interesting issue, also explored by Kritsuk et al. (2007) relates to the power spectrum of the *corrected* velocity, $\rho^{1/3} v$. This quantity stems for the fact that the energy flux which for incompressible turbulence is simply $\propto v^3$ becomes for compressible fluids ρv^3. Interestingly, Kritsuk et al. (2007) find that the power spectrum of this quantity has an exponent much closer to 11/3 than the power spectrum of v. This raises the question as to whether the ideas of Kolmogorov can be generalized and apply to compressible flow as well.

Density is an important quantity to characterize in compressible flows. It is also of fundamental importance for many astrophysical problems. Numerical simulations of supersonic turbulence for which the driving is performed in the solenoidal modes, have established that the density PDF is typically lognormal, i.e. $\log \rho$ has a normal distribution (e.g. Vázquez-Semadeni, 1994 ; Padoan et al. 1997). The width of the distribution has been established to be $\sigma^2 = \log(1 + b^2\mathcal{M}^2)$ where b is a constant of the order of $\simeq 0.5$ (Federrath et al. 2008). Thus as the Mach number is increased, the distribution broadens, implying that the quantity of dense gas increases. This lognormal distribution plays an important role in theoretical calculations related to the star formation problems (Krumholz & McKee, 2005 ; Hennebelle & Chabrier 2008). Another important quantity to compute is the power spectrum of ρ since it characterizes the size distribution of the density fluctuations. In the subsonic case, the density power spectrum is found to have an index very close to the Kolmogorov value (Kim & Ryu, 2005). In the supersonic case, the spectrum becomes gradually flatter up to very shallow index. This is probably due to the fact that in supersonic flows shocked sheets with very stiff boundaries form. Since they are mathematically equivalent to Dirac function whose Fourier transform is $\propto k^0$, this produces very shallow powerspectrum. A more meaningful quantity to work with, is $\log \rho$ whose powerspectrum turns out to be close to the 11/3 value even in the supersonic case. A link between the power spectrum and the clump mass spectrum has been proposed by Hennebelle & Chabrier (2008) who establish that the exponent, γ, of the mass spectrum is linked to the exponent, n, of the power spectrum of $\log \rho$ through the relation: $\gamma = -2 + (n - 3)/3$. For $n = 11/3$, one obtains $\gamma \simeq -1.8$. This values turns out to be compatible with what has been inferred in observations (Heithausen et al., 1998) and also in numerical simulations (Audit & Hennebelle 2010).

Recently, it has been pointed out by Federrath et al. (2010) that the way turbulence is driven has important consequences on the flow properties. In particular, they investigate purely solenoidal and purely compressible forcing. They find that the statistics are significantly different. In particular, the density PDF is not exactly a lognormal distribution in the compressible case developing a high density tail. The index of the various powerspectra are also different. As for the MHD case, this raises the question as to whether the compressible turbulence is truly universal. From an astrophysical point of view, the impact of the forcing gives raise to the question of its origin and nature. This also raises the question of how the clouds is defined and connected to the surrounding medium. To address this last point, it is necessary to go beyond the simple isothermal assumption and treat the proper thermodynamics of the interstellar gas. Note that Passot & Vázquez-Semadeni (1998) have shown performing 1D simulations that γ, the adiabatic exponent, has a drastic influence on the density PDF. Indeed, as γ decreases below 1, the high density part of the PDF becomes a power law, whose exponent becomes gradually shallower, rather than an exponential. This has been confirmed by 3D simulations (Audit & Hennebelle 2010).

Compressible isothermal turbulence has also been recently investigated (e.g. Padoan & Nordlund, 1999 ; Vestuso et al., 2003 ; Beresnyak et al., 2005) mainly by numerical simulations. It entails a new number which is usually called β and is equal to the ratio of the thermal over magnetic pressure. Low β plasma are dominated by the magnetic energy. Although different by many aspects to the hydrodynamical case, similar features are found, including strong density fluctuations in the supersonic case, power spectrum inbetween the Kolmogorov and the Burgers one. An important aspect having wide astrophysical implication is the correlation between magnetic intensity and density. Passot & Vázquez-Semadeni (2003) show numerically and analytically that density and magnetic intensity tend be anti-correlated in a slow MHD wave while they tend to be correlated in a fast MHD wave. Thus in subalfvénic flows, density and magnetic field tend to be anti-correlated and correlated in supersonic ones.

4.2. Turbulence in multi-phase flows

As described above, the ISM presents various phases, meaning that at a given pressure, the gas can be in a variety of thermodynamical states. To treat the ISM properly, it is therefore necessary to solve an energy equation and compute the heating and cooling that the fluid particles undergo.

As one may anticipate, it is even harder to get exact results regarding the turbulence properties in this context. Indeed, unlike in incompressible and isothermal turbulence, such a flow is not described only by few numbers, but depends on cooling and heating processes which can be complex leading sometimes to qualitatively new behaviours. One of the most remarkable is certainly the existence of thermally unstable states (Field, 1965) which gives raise to the phases. Basically, thermal instability occurs when the condition $\frac{\partial P}{\partial \rho})_{\mathcal{L}} < 0$ is fulfilled where $\mathcal{L}$ is the net loss function describing the cooling and heating by radiative processes. Physically this means that considering density fluctuations, the pressure is dropping when the density increases implying that the piece of gas is further compressed by the external pressure. In a 2-phase flow, at a given pressure, the gas can be in two different states, one diffuse and warm and one dense and cold (e.g. Wolfire et al., 1995). The two phases are connected by stiff thermal fronts whose length, called the Field length, is given by an equilibrium between the cooling function and the thermal diffusivity. Various new scales have thus to be considered such as the cooling length of the WNM which is the product of the sound speed and the cooling time and the above mentioned Field's length. Recent studies have shown that compressible motions arising in turbulent flows could induce dynamically the phase transition leading to the formation of cold clouds embedded in a warm confining phase (Hennebelle & Pérault, 1999 ; Koyama & Inutsuka, 2000 ; Sanchez-Salcedo et al., 2002). In 2D and 3D this gives raise to a state that combines on one hand the picture of classical turbulent flow but on the other hand presents cold structures embedded in warm gas and connected by stiff thermal fronts (Koyama & Inutsuka, 2002 ; Audit & Hennebelle, 2005 ; Heitsch et al. 2006). The cold structures are found to typically have a velocity dispersion of the order of the sound speed of the warm phase in which they are embedded. Since their internal sound speed is about 10 times lower than the sound speed of the warm phase, this implies that collisions at typically Mach ten are arising leading to further density enhancements. The statistics of the cold structures has been studied by Hennebelle & Audit (2007). One striking aspect, is that the mass spectrum of the CNM structures is very similar to the one inferred for the clumps in isothermal gas. This suggests that turbulence is seeding the formation of structures and that their masses reflect the mass of the gas within the perturbations.

Interestingly, thermally unstable gas, which has been inferred from observations (Heiles 2001) has also been found in numerical simulations by Gazol et al. (2001) and others. Audit & Hennebelle (2005) have shown that as turbulence is increased, the fraction of thermally unstable gas increases at the expense of the cold gas. They also propose that the shear is able to stabilize partially the gas against thermal instability. Generally speaking, the abundances of the various phases significantly depend on the dynamics of the flow (Seifrid et al. 2011).

The various power spectra and structure functions which have been measured in 2-phase flows (Kritsuk & Norman, 2004 ; Hennebelle & Audit, 2007) are relatively similar to those measured in standard isothermal flows although numerical resolution is possibly an issue given the large range of scales involved in this problem.

The influence of the magnetic field in such a flow has also been investigated. While Field (1965) considering a simple plane parallel configuration with purely transverse magnetic field, suggests that strong magnetic fields can suppress thermal instability (because the total pressure, P_T, i.e. thermal and magnetic pressures, can be such that $\frac{\partial P_T}{\partial \rho})_{\mathcal{L}} > 0$ even if $\frac{\partial P}{\partial \rho})_{\mathcal{L}} < 0$), Hennebelle & Pérault (2000) and Inoue & Inutsuka (2008) show that if the magnetic field is not purely transverse, magnetic tension can channel the growing perturbation along the field lines.

The above mentioned studies have been performed at relatively small scales (typically few tens of parsec). Attempting to simulate a representative part of the Galaxy, simulations have been performed at much larger scales ($\simeq$1 kpc) by de Avillez & Breitschwerdt (2005) and Joung & Mac Low (2006). In these works, the three phases (HIM, WNM, CNM) are treated and the supernovae have been introduced, thus consistently driving the turbulence. The vertical structure of the gas is well described and gives raise to galactic fountains and chimneys. The magnetic field is also self-consistently treated. It is found to have a complex and tangled structures. Another interesting aspects is that these simulations can reproduce many observed features of the diffuse ISM.

References

Audit, E., Hennebelle, P., 2005, A&A, 433, 1
Audit, E., Hennebelle, P., 2010, A&A, 511, 76
de Avillez, M., Breitschwerdt, D., 2005, A&A, 436, 585
Beresnyak et al., ApJ, 2005, 624, 93
Biskamp, D., 2003, matu.book
Boldyrev, S., 2005, ApJ, 626, L37
Boldyrev, S., 2006, PhRvL, 97, 550
Cho, J., Lazarian, A., Vishniac, E., 2002, ApJ, 564, 291
Elmegreen, B., Scalo, J., 2004, ARA&A, 42, 211
Federrath, C., Klessen, R., Schmidt, W., 2008, ApJ, 688, 79
Federrath, C., et al. 2010, A&A, 512, 81
Field, G., 1965, ApJ, 142, 131
Gazol, A. et al. 2001, ApJ, 557, 121
Goldreich, P., Sridhar, S., 1995, ApJ, 438, 763
Heiles, C., 2001, ApJ, 551L, 105
Heiles, C., Troland, T., 2005, ApJ, 624, 773
Heithausen et al., 1998, A&A, 331, 65
Heitsch, F. et al. 2006, ApJ, 648, 1052
Hennebelle, P., Pérault, M., 1999, A&A, 351, 309
Hennebelle, P., Pérault, M., 2000, A&A, 359, 1124
Hennebelle, P., Audit, E., 2007, A&A, 465, 431
Hennebelle, P., Chabrier, G., 2008, ApJ, 684, 395
Inoue, T., Inutsuka, S.-i., 2008, ApJ, 687, 303
Iroshnikov, P., 1963, Sov. Astron. 7, 566,
Joung, M., Mac Low, M.-M., 2006, ApJ, 653, 1266
Kim, J., Ryu, D., 2005, ApJ, 630, 689
Koyama, H., Inutsuka, S.-i., 2000, ApJ, 532, 980
Koyama, H., Inutsuka, S.-i., 2002, ApJ, 564, 97
Kraichnan, R. 1965, Phys. Fluids, 8, 1385,
Kritsuk, A., Norman, M., 2004, ApJ, 601, 55
Kritsuk, A., Norman, M., Padoan, P., Wagner, R., 2007, ApJ, 665, 416
Kolmogorov, A., 1941, Dokl. Akad. Nauk., SSSR, 31, 538
Krumholz, M., McKee, C., 2005, ApJ, 630, 250
Larson, R., 1981, MNRAS, 194, 809
Lee, E., Brachet, M.-E., Pouquet, A., Mininni, P., Rosenberg, D., 2010, PhRvE, 81, 6318L
Mac Low, M.-M., Klessen, R., 2004, RvMP, 2004, 76, 125
McKee, C., Ostriker, E., 2007, ARA&A, 45, 565
Monin, A., Yaglom, A., 1975, Mechanics of turbulence, vol 2, MIT, Press, Cambridge MA
Padoan, P., Nordlund, A., Jones, B., 1997, MNRAS, 288, 145
Padoan & Nordlund, 1999, ApJ, 526, 279
Passot, T., Vázquez-Semadeni, E., 1998, Phys Rev, 58, 4501
Passot, T., Vázquez-Semadeni, E., 2003, A&A, 398, 845
Sanchez-Salcedo, F., Vázquez-Semadeni, E., Gazol, A., 2002, ApJ, 577, 768
She, Z.-S., Leveque, E., 1994, Phys. Rev. 72, 336
Seifrid, D., Schmidt, W., Niemeyer, J., 2011, A&A, 526, 14
Vázquez-Semadeni, E., 1994, ApJ, 423, 681
Vestuso et al. 2003, ApJ, 590, 858
Wolfire, M., McKee, C., Hollenbach, D., Tielens, A., 1995, ApJ, 453, 673