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Memorie della

Gravitational lens

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Abstract. After some historical remarks, we consider observational data on the gravitational lensing, different types of lensing: strong, weak, and microlensing, discovery of planets around distant stars by microlensing. We consider lensing with large deviation angles, when light passes close to the gravitational radius of the lens, and formation of weak relativistic rings. In the last part we consider an influence of plasma on the gravitational lensing. When a gravitating body is surrounded by a plasma, the lensing angle depends on the frequency of the electromagnetic wave, due to dispersion properties of plasma, in presence of a plasma inhomogeneity, and due to a gravity. The second effect leads, even in a uniform plasma, to a difference of the gravitational photon deflection angle from the vacuum case, and to its dependence on the photon frequency. Both effects are taken into account. Dependence of the lensing angle on the photon frequency in a homogeneous plasma resembles the properties of a refractive prism spectrometer, which strongest action is for longest radiowaves. We have shown that the gravitational effect could be detected in the case of a hot gas in the gravitational field of a galaxy cluster.

Key words. General relativity. Gravitational lens.

1. Introduction

General Relativity predicts that a light ray which passes by a spherical body of mass M with impact parameter b, is deflected by the Einstein angle:

$$\hat{\alpha} - \frac{4GM}{c^2 b} = \frac{2R_s}{b}, \qquad (1)$$

provided the impact parameter b is much larger than the corresponding Schwarzschild radius R_S:

Long time ago Newton asked a rhetorical question: Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action strongest at the least distance?, with an evident positive answer. In newtonian gravitational theory the deviation angle was calculated by (Soldner 1801), with an answer $\alpha = 2GM/(c^2 b)$. In general relativity the deviation angle occurs to be, according to Einstein (1915), two times larger (1). For the light deviation by the Sun the Einstein theory gave the value of deviation angle $\alpha = 1.75$". Note, that before creation of general relativity, Einstein (1911) had obtained the Newtonian value of the deflection angle in a special relativity. To check these predictions the expedition was installed, headed by A. Eddington, to measure the positions of stars in the Hyades

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Fig. 1. Observations during Solar Eclipse, from http://www.uwsp.edu/physastr/kmenning/Phys300/Lect10.html.

constellation around the Sun, during the solar eclipse. The expedition of 29 May 1919 year confirmed the Einstein result, Dyson et al. (1920), what, together with the precession of the Mercury orbit, and measurements of the red shift, was one of 3 proofs of a validity of General Relativity, see fig.1.

The development of the theory of gravitational lensing in the pre second world war period was connected with names of Lodge (1919), who invented the term "gravitational lens", Eddington (1920), who had shown the possibility of observations of multiple images of one lensed source, Chwolson (1924), who had found that a lensed image may have a circular form (ring), when the source, lens and observer are on the same strait line, . The magnification of a lensed image was investigated by Tihov (1937), and (Zwicky 1937a) had shown, that the gravitational lensing on the massive extragalactic "nebulae" (present galaxies or clusters of galaxies) is much more effective than on stars. The estimations of the probability of "Detecting Nebulae Which Act as Gravitational Lenses" have led Zwicky (1937b) to the conclusion that "The probability that nebulae which act as gravitational lenses will be found becomes practically a certainty". This conclusion he have done after a sceptical remark of Einstein (1936): "Of course, there is not much hope of observing this phenomenon directly". In the same paper he noticed that among multiple there is one, the most brightest the biggest cat gets all the milk. Discovery of the first gravitational lens was done in 1979 by Walsh et al. (1979) in the observation of the quasar with two images 0957 + 561 A, B. The authors claimed that it could be a twin quasistellar objects or gravitational lens. The second suggestion happens to be correct, see fig.2.

Well before this discovery the theory of gravitational lensing was developed in the papers of Klimov (1963), Liebes (1964), Refsdal (1964), Ingel' (1974). After discovery of quasars it become clear that these objects are ideal sources for a search of gravitational lensing, and that was confirmed by observations Walsh et al. (1979). The first international conference, which took place in 1983, was centered on quasars, Swings (1984). The idea of microlensing, and calculations of the corresponding light curve was done by Byalko (1969). Observations of the microlensing events have been started in 1991, and the first event was discovered by Alcock, et al. (1993) and Aubourg, et al. (1993), in the experiments observing Large Magellanic Cloud, and the Galactic halo, respectively.

2. Gravitational lensing in vacuum

In the most astrophysical situations related with gravitational lensing approximation of weak deflection with $b \gg R_s$ is well satisfied. This angle does not depend on frequency of the photon

2.1. Observational data

The most picturesque gravitational lens have their own names.

1°. Einstein Ring in the gravitational lens system B1938 + 666, King et al. (1998). The angular diameter of this ring is about 1 arc sec, and the liner diameter is 10^4 parsec.

2°. The Cloverleaf Quasar (H1413+117). The Hubble optical image of Cloverleaf Quasar is on the cite `http://chandra.`

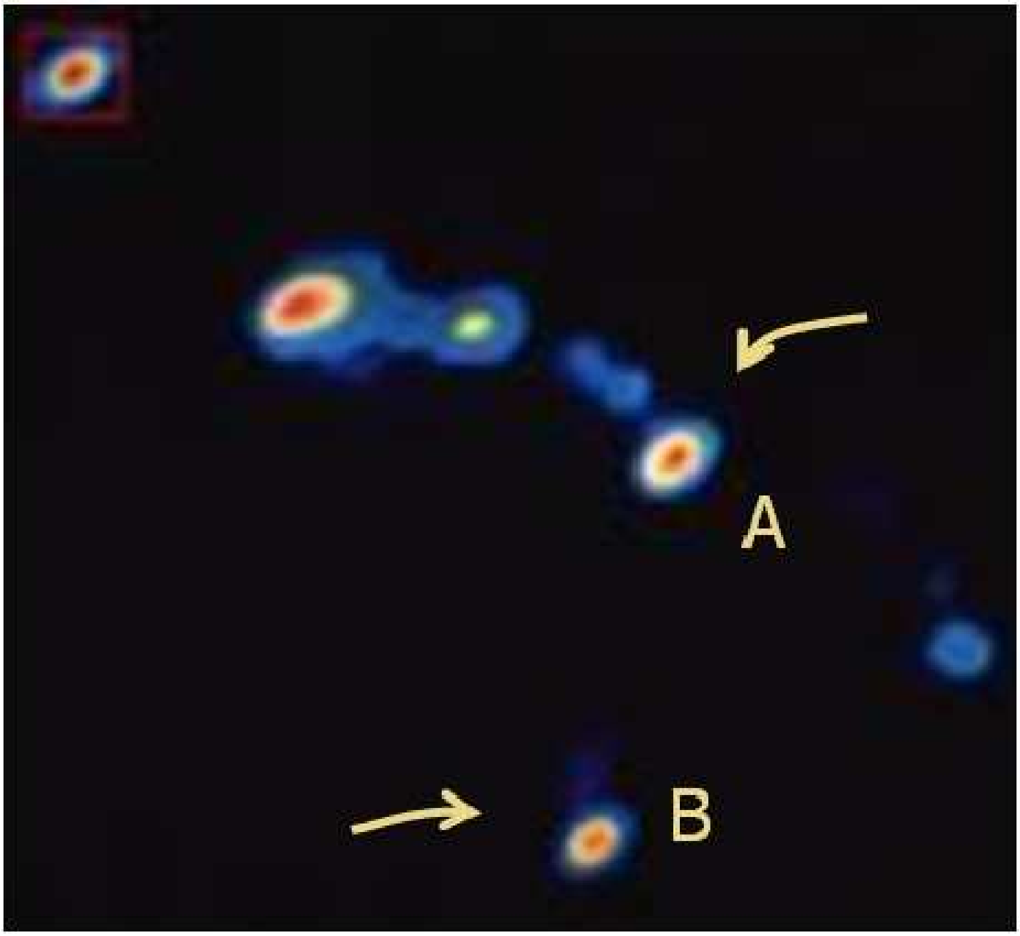

Fig. 2. The first observed lense (1981): QSO 0957+561, maximum separation - 6.1 arcsec, Image redshift - 1.41, Lens redshift - 0.36, B/A = 2/3. Radio map of the double quasar (0957+561), at 6 cm wavelength, taken with the Very Large Array (VLA) radio telescope, New Mexico. These two images of a single distant quasar are the result of the gravitational bending of light from the quasar by an intervening body. Image A is the bright point source (top) emitting radio jets on either side. Image B is the bright point-like object below A; the jets have not been doubly imaged. The weak image just above B coincides with the center of a large galaxy observed in the optical, which, along with the rich cluster of galaxies it lies in, is believed to be acting as the gravitational lens. Observers; P. Greenfield, D. Roberts & B. Burke. Taken 13/Oct/79. From http://www.sciencephoto.com/media/333974/enlarge.

`harvard.edu/photo/2004/h1413/more.html`

3°. The Einstein Cross, Q2237+0305, Huchra, et al. (1985). Four QSO images arrayed around the nucleus of the galaxy are observed. The model of this object was constructed by Schneider, et al. (1988). The picture of the gravitationally lensed quasar Q2237+0305 and the associated lensing spiral galaxy was taken by the 3.5-meter WIYN telescope, on the night of October 4, 1999, see `http://antwrp.gsfc.nasa.gov/apod/ap070311.html`

4°. Double Einstein ring around the gravitational lens RDSSJ0946+1006, Gavazziet al. (2008). The main lens is at redshift z_l=0.222,

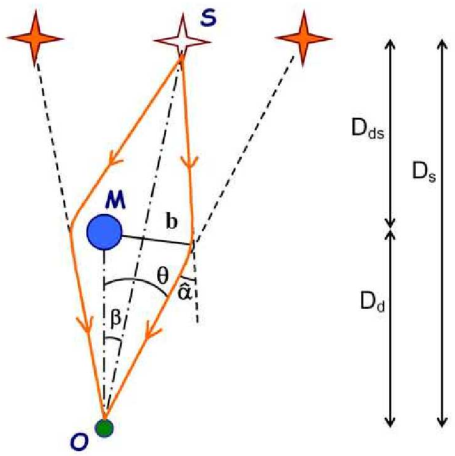

Fig. 3. On basis of Einstein deflection angle ordinary GL theory is developed. At this picture there is the example of the simplest model of Schwarzschild point-mass lens which gives two images of source instead of one single real source.

while the inner ring is at redshift z_{s1}=0.609. The angular radius of the outer ring 2 is about 2.07", what is $\sim$ 1.5 times larger than the radius of the inner ring 1. That implies a higher redshift than that of ring 1; the detection of ring 2 in the F814W ACS filter implies an upper limit of $z_{s2} \lesssim 6.9$.

2.2. Image form and magnification factor

The surface brightness I for an image is identical to that of the source in the absence of the lens. The flux of an image of an infinitesimal source is the product of its surface brightness and the solid angle $\Delta\omega$ it subtends on the sky. The magnification factor μ is the ratio of the flux of the image to the flux of the unlensed source, and is equal to the ratio of the solid angles of the image, and the unlensed source

$$\mu = \frac{\Delta\omega}{(\Delta\omega)_0} \tag{2}$$

From the observational point of view there are three types of the gravitational lensing.

1°. Strong lensing. This type of lensing is characterized by multiple images, what, together with spectral and time variation properties of the images, permits to obtain a clear conclusion about the presence of the gravitational lens. All above examples belong to "strong lensing" cases.

2°. "Weak lensing". The lensing images are characterized by weak distortions and small magnifications. Usually only one image is visible. Those cannot be identified in individual sources, but only in a statistical sense. The example of a weak lensing is given in fig.4. Observations of many distorted images of galaxies give a possibility to reconstruct the mass distribution in the gravitational lens. The example of the reconstruction of the distribution of the dark matter gravitational potential in merging Bullet cluster of galaxies 1E0657-558 (z=0.296), by HST/ASC observation of the field galaxies behind the clusters, is given in the paper of Clowe et al. (2006). This distribution is obtained together with the distribution of ordinary matter, by Chandra X-ray observations of the shocks formed in the merging cluster 1E0657-558, which do no coincide with the maxima of the dark matter gravitational potential. This observations are considered as a direct empirical proof of existence of dark matter.

3°. Microlensing. Version of strong lensing in which the image separation is too small to be resolved. Only change of the flux is visible. The term microlensing (ML) arose from the fact that a single star of about solar mass can lead to images split into microimages separated by microarcseconds. Multiple images due to stellar-mass objects can therefore not be resolved; the only observable effect ML is the alteration of the apparent luminosity of background sources. The images of a multiply imaged QSO are seen through a galaxy; since galaxies contain stars, stellar-mass objects can affect the brightness of these images. A very interesting example of this effect is a microlensing on planets. Data from a microlensing event indicate a smooth, symmetric magnification curve as a lens star moves between a source star and an observatory on Earth. The short

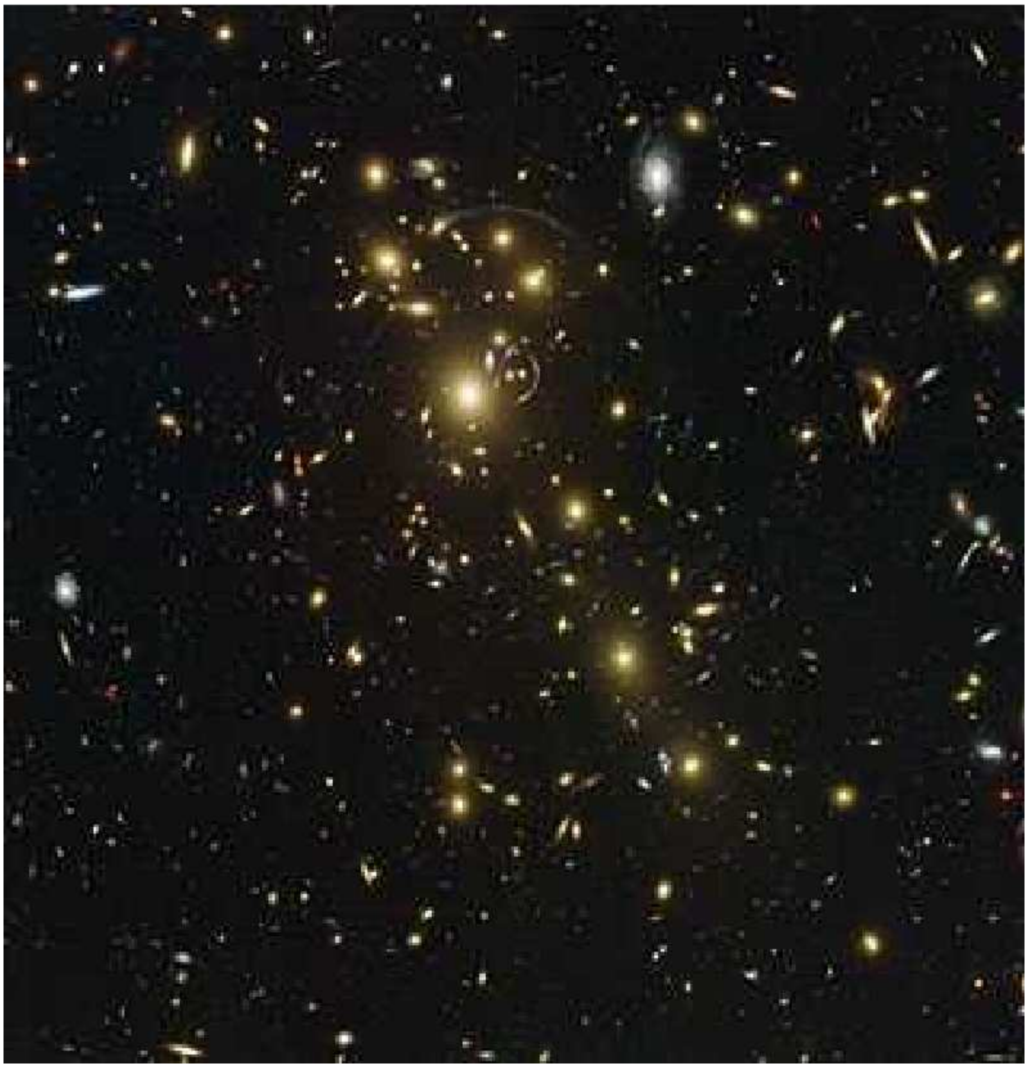

Fig. 4. Located in the northern celestial hemisphere, Abell 1703 is composed of over one hundred different galaxies that act as a powerful cosmic telescope, or gravitational lens. The gravitational lens produced by the massive galaxy cluster in the foreground (the yellow mostly elliptical galaxies scattered across the image) bends the light rays in a way that can stretch the images and so amplify the brightness of the light rays from more distant galaxies. In the process it distorts their shapes and produces multiple banana-shaped images of the original galaxies. The result is the stunning image seen here - a view deeper into the Universe than possible with current technology alone. Abell 1703 is located at 3 billion light-years from the Earth (z = 0.26), from the paper of Saha and Read (2009).

spike in magnification is caused by a planet orbiting the lens star, see fig.5. The analysis permits to obtain estimations about the mass of the planet, and its orbit.

2.3. Small and large deflection angle

From a physical point of view a weak deflection denote a situation, when a deflection angle

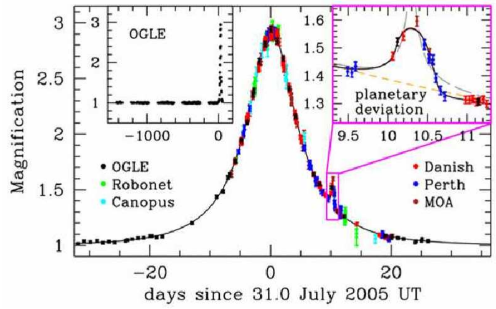

Fig. 5. Microlensing of the planet. Light curve of the OGLE-2005-BLG-390, from http://www.centauri-dreams.org/?p=520,

is small. General Relativity predicts that a light ray which passes by a spherical body of a mass M, with an impact parameter b, is deflected by the Einstein angle (1), provided the impact parameter b is much larger than the corresponding Schwarzschild radius $R_S = 2GM/c^2 \ll b$.

In the Schwarzschild metric the equations for the plane photon motion, in the polar coordinate system (r, φ) is written in Bisnovatyi-Kogan, Tsupko (2008), as

$$\left(\frac{dr}{d\lambda}\right)^2 + B^{-2}(r) = b^2, \tag{3}$$

$$\frac{d\varphi}{d\lambda} = \frac{1}{r^2}, \quad \frac{dt}{d\lambda} - \frac{1}{b}\left(1 - \frac{2M}{r}\right)^{-1}.$$

The type of the solution of (3) depends on the value of the impact parameter b.

1°. If $b < 3\sqrt{3}M$, then the photon falls to $R_S = 2M$ and is absorbed by the black hole.

2°. If $b > 3\sqrt{3}\,M$, then the photon is deflected by an angle $\hat{\alpha}$ and flies off to infinity. Here there are two possibilities:

2a°. If $b \gg 3\sqrt{3}M$, then the orbit is almost a straight line with a small deflection by an angle $\hat{\alpha} = 4M/R$, where R is the distance of closest approach. This is the case customarily examined in the theory of weak gravitational lensing, when the impact parameter is much greater than the Schwarzschild radius of the lens.

2b°. If $0 < b/M - 3\sqrt{3} \ll 1$, then the photon makes several turns around the black hole near a radius $r = 3M$ and flies off to infinity.

The case when the deflection angle is large may be called as a "strong lensing". The exact value of the deflection angle is determined as

$$\hat{\alpha} = 2\int_R^\infty \frac{dr}{r^2}\left[\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)\right]^{-1/2} - \pi. \tag{4}$$

Here R is the distance of the closest approach, b is the impact parameter, M is the mass of the black hole.

In the strong deflection limit, if the value of the impact parameter is close to the critical value $b \gtrsim 3\sqrt{3}$, $R \gtrsim 3$, we can use approximate analytical formula for the deflection angle in strong deflection limit from Bozza et al. (2001), Bozza (2002)

$$\hat{\alpha} = -2\ln\frac{R - 3M}{36(2 - \sqrt{3})M} - \pi\,, \tag{5}$$

or

$$\hat{\alpha} \simeq -\ln\left(\frac{b}{b_{cr}} - 1\right) - 0.40023. \tag{6}$$

Relation between impact parameter b and distance of the closest approach R is written as

$$b^2 = \frac{R^3}{R - 2M}, \tag{7}$$

The formulae (5),(6) are for the case when the incident photon from infinity undergoes one or more revolutions around the black hole and then escapes to infinity.

Let the source, lens, and observer lie on one straight line (fig.6). The theory of gravitational lensing shows that in this case a circle, known as the Einstein ring, is formed Schneider, Ehlers & Falco (1992). Inside this "main" ring there are rings formed by photons which have been deflected by 2π, 4π, 6π ... ; these rings are sometimes referred to as relativistic rings, Virbhadra and Ellis (2000).

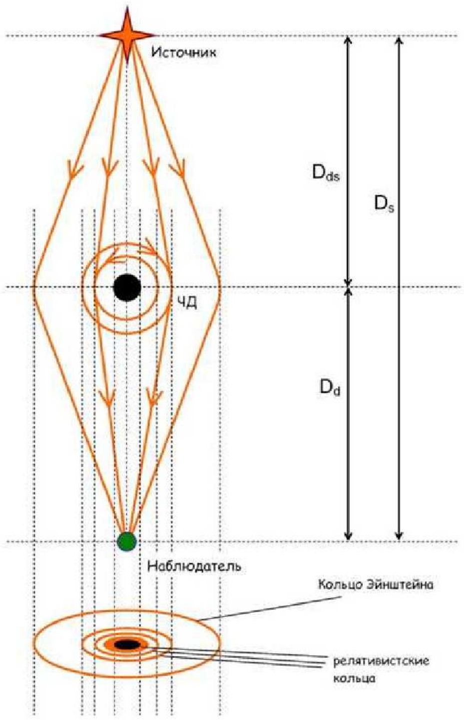

Fig. 6. Einstein ring, and relativistic rings

Equating $\hat{\alpha}$ to 2π, 4π, 6π ... , we find that these relativistic rings are localized at the impact parameters, see Misner et al. (1973):

$$b/M - 3\sqrt{3} = 0.00653,\ 0.0000121,$$

$$0.0000000227,\ 0.423 \cdot 10^{-10},\ 0.791 \cdot 10^{-13}\ \ldots .$$

The values of the impact parameters can also be obtained using the approximation of the strong deflection, i.e. equation (6). Using the equation $\hat{\alpha}(b) = 2\pi n$ ($n = 1, 2, 3...$), we obtain the following values of b_n:

$$b_n = b_{cr}(1 + e^{C_1 - 2\pi n}),\ C_1 = -0.40023\,. \qquad (8)$$

The amplification factor for the relativistic rings is much smaller that for the Einstein ring, and is much smaller that unity. These factors, have been calculated by Bisnovatyi-Kogan, Tsupko (2008). For a distant quasar with $M_* = 10^9 M_\odot$, $R_* = 15 R_{*S}$ ($R_{*S} = 2GM_*/c^2$), $D_{ds} = 10^3$ Mpc, $D_d = 3$ Mpc, $D_s \approx D_{ds}$, and a lens of mass $M = 10^7 M_\odot$, the amplification factors are

$$\mu_0 = 1 \cdot 10^6,\ \mu_1 = 2 \cdot 10^{-15},\ \mu_2 = 4 \cdot 10^{-18}\,. \qquad (9)$$

Here μ_0, μ_1 and μ_2 are amplification factors for the Einstein ring, the first and the second relativistic rings, respectively.

3. Gravitational lensing in plasma

Theory of gravitational lensing is developed for the light propagation in vacuum, see Schneider, Ehlers & Falco (1992), Schneider, Kochanek & Wambsganss (2006). The photon trajectories and deflection angles in vacuum don't depend on the photon frequency (or energy). Therefore gravitational lensing in vacuum is achromatic. For example, in Schwarzschild point-mass lens two images have the same complicated spectra, formed by the photons of different frequencies which undergo deflection by the same angle.

It is interesting to consider the gravitational lensing in a plasma, because in space the light rays mostly propagate through this medium. When we work in the frame of geometrical optics what is usual for gravitational lensing, we can characterize properties of medium by using of the refractive index. In an inhomogeneous medium (the refraction index depends on space coordinates) a photon moves along the curved trajectory. If medium is also dispersive (the refraction index depends on the photon frequency) the trajectory depends on the photon frequency. The effect of refractive deflection of light in the inhomogeneous medium has no relation to gravity.

In a medium the light rays move with the group velocity. For plasma with index of refraction $n = 1 - \frac{\omega_e^2}{\omega^2}$ (ω_e^2 is the plasma electron frequency, ω is the frequency of the photon) we have the relations specific for plasma: the phase velocity is $v_{phase} = c/n$, the group velocity is $v_{group} = cn$, $v_{phase} v_{group} = c^2$.

Deflection of light rays due to action of both gravity and plasma is not new problem. This problem was considered in some works, in application to gravitational lensing it was discussed in details by Bliokh, Minakov (1989) where gravitational lensing by gravitating body with surrounding spherically symmetric plasma distribution was considered.

The consideration usually was done in a linear approximation, when the two effects: the vacuum deflection due to the gravitation (Einstein angle), and the deflection due to the non-homogeneity of the plasma, have been considered separately. The first effect is achromatic, the second one depends on the photon frequency because plasma is dispersive medium, but it is equal to zero if the plasma is homogeneous.

A general theory of the geometrical optic in the curved space-time, in arbitrary medium, is presented in the book of Synge (1960). On the basis of his general approach we have developed the model of gravitational lensing in plasma and obtained some new features of the problem, Bisnovatyi-Kogan, Tsupko (2009), Bisnovatyi-Kogan, Tsupko (2010).

We have considered static weak gravitational field and plasma with refractive index

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(x^\alpha)]^2}, \tag{10}$$

$$\omega_e^2 = \frac{4\pi e^2 N(x^\alpha)}{m} = K_e N(x^\alpha). \tag{11}$$

Here $\omega(x^\alpha)$ is the frequency of the photon, which depends on the space coordinates x^1, x^2, x^3 due to the presence of the gravitational field (gravitational red shift). We denote $\omega(\infty) \equiv \omega$, e is the charge of the electron, m is the electron mass, ω_e is the electron plasma frequency, $N(x^\alpha)$ is the electron concentration in inhomogeneous plasma.

We have shown for the first time that due to dispersive properties of plasma even in the homogeneous plasma gravitational deflection will differ from vacuum deflection angle, and gravitational deflection angle in plasma will depend on frequency of the photon:

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right). \tag{12}$$

The presence of plasma increases the gravitational deflection angle. This formula is valid under the condition of smallness of $\hat{\alpha}$, but this condition allows the second term in brackets to be much larger than the first one. So, the gravitational deflection in plasma can be significantly larger than in the vacuum. This formula is valid only for $\omega > \omega_e$, because the waves with $\omega < \omega_e$ do not propagate in the plasma. Under $\omega_e = 0$ (concentration $N(x^\alpha) = 0$) or $\omega \to \infty$ this formula turns into the deflection angle for vacuum $2R_S/b$.

In homogeneous plasma photons of smaller frequency, or larger wavelength, are deflected by a larger angle by the gravitating center. The effect of difference in the gravitational deflection angles is significant for longer wavelengths, when ω is approaching ω_e. That is possible only for the radio waves. Therefore, the gravitational lens in plasma acts as a radiospectrometer, Bisnovatyi-Kogan, Tsupko (2009).

We should use formula (12) when we consider gravitational lensing of radiowaves by point or spherical body in presence of homogeneous plasma. This effect has a general relativistic nature, in combination with the dispersive properties of plasma. We should also emphasize that the plasma is considered here as the medium with a given index of refraction, and this formula does not take into account self-gravitation of plasma. Our calculations show that this effect takes place only if the medium is dispersive. In a homogeneous medium without dispersion, the gravitational deflection is equal to the vacuum gravitational deflection, although the group velocity of light in the medium is less than in the vacuum Bisnovatyi-Kogan, Tsupko (2009).

We would like to mention the book of Perlick (2000), where similar approach for the gravitational deflection of light in plasma

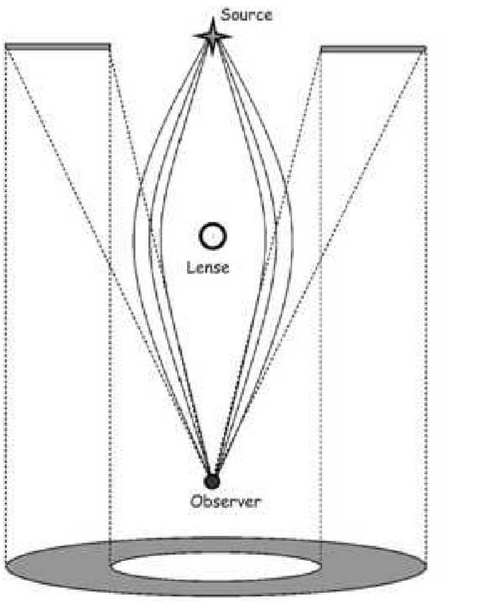

Fig. 7. Instead of a ring with complicated spectra, we will have a thick strip, formed by the photons with different frequency, which is decreasing with increasing of the radius,of the ring.

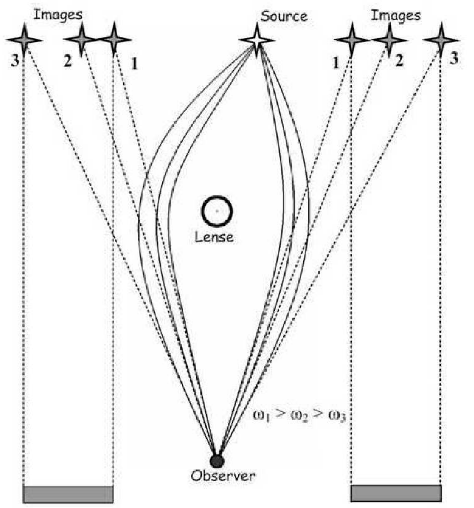

Fig. 8. Instead of two concentrated images with complicated spectra, we will have two line images, formed by the photons with different frequencies, which are deflected by different angles, which are increasing with decreasing of the frequency.

was developed. The general formula for a light deflection in the Schwarzschild metric, in the presence of plasma is obtained in the form of the integral, but particular cases of the homogeneous plasma, and different lens models are not considered, and the estimation of the difference of the gravitational deflection in plasma, from the vacuum case, is not obtained.

The observational effect of the frequency dependence may be represented on the example of the Schwarzschild point-mass lens. Instead of two concentrated images, or a ring, with complicated spectra, we will observe two line 'rainbow' images, or a thick ring, formed by the photons with different frequencies, which are deflected by different angles, see figs.7,8.

We can estimate an angular difference of positions of images in different bands due to effect of gravitational radiospectrometer. The angular half separation due to gravitational lensing, between the images of the source in vacuum (Schneider, Ehlers & Falco 1992) is of the order of

$$\theta_0 = \sqrt{2R_S \frac{D_{ds}}{D_d D_s}}\,, \tag{13}$$

where D_d is the distance between the observer and the lens, D_s is the distance between the observer and the source, D_{ds} is the distance between the lens and the source. Lensing in presence of a homogeneous plasma (12) leads to an angular half separation between images as

$$\theta_0^{pl} = \sqrt{\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right) R_S \frac{D_{ds}}{D_d D_s}} =$$

$$= \theta_0 \sqrt{\frac{1}{2}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)}\,, \tag{14}$$

which may be called, as plasma Einstein ring. For $\omega_e^2/\omega^2 \ll 1$ we obtain

$$\theta_0^{pl} = \left(1 + \frac{1}{4}\frac{\omega_e^2}{\omega^2}\right)\theta_0 \,. \qquad (15)$$

The difference between angular separations of images in vacuum and in plasma $\Delta\theta_0$, produced by the same lens configuration, is equal to

$$\frac{\Delta\theta_0}{\theta_0} = \frac{\theta_0^{pl} - \theta_0}{\theta_0} = \frac{1}{4}\frac{\omega_e^2}{\omega^2} \simeq 2.0 \cdot 10^7 \,\frac{N_e}{\nu^2}, \qquad (16)$$

where ν is the photon frequency in Hz, $\omega = 2\pi\nu$. The formula (16) gives the difference between positions of the radio image with a frequency ν, and the optical image, which may be described by the vacuum formula (13).

As an example, let us estimate a possibility of the observation of this effect by the planned project Radioastron, see http://www.asc.rssi.ru/radioastron/index.html. The Radioastron is the VLBI space project led by the Astro Space Center of Lebedev Physical Institute in Moscow. For the lowest frequency of the Radioastron, $\nu = 327 \cdot 10^6$ Hz, the angular difference between the vacuum and the plasma images is about 10^{-5} arcsec, when the plasma density on the photon trajectory, in the vicinity of the gravitational lens is of the order of $N_e \sim 5 \cdot 10^4$ cm^{-3}. This angular resolution is supposed to be reached in the project Radioastron.

Gravitational lensing also leads to magnification of source. This means that the flux of image is bigger or smaller than the flux of source, different images have different magnifications. In observations we don't know the intrinsic flux of source and its spectrum, but we can compare the fluxes of images in different frequencies. The ratios of the fluxes of images in different bands should be the same, if we consider lensing in vacuum, because gravitational deflection in vacuum is achromatic.

The magnification is determined by the deflection law, (Schneider, Ehlers & Falco 1992). In the case of radio wave lensing in plasma we have another formula for the deflection instead of the Einstein angle, so formulas for the magnification will be different. It leads to difference of magnigication factors of different images in different bands, when the light propagates in regions with different plasma density. By-turn it leads to difference of ratios of the fluxes of images in different bands (see details in Bisnovatyi-Kogan, Tsupko 2010). It is another prediction of model of gravitational radiospectrometer for observation. We should mention that Thompson scattering and absorption during propagation of radiation through the plasma can significantly change flux and complicate the phenomena of lensing magnification.

Discussion above is for the Schwarzschild lens in the homogeneous plasma. In a case of a plasma non-homogeneity there is the refractive deflection. Our approach allows us to consider two effects simultaneously: the gravitational deflection in plasma which is different from the Einstein angle (new effect), and the nonrelativistic effect (refraction) connected with the plasma inhomogeneity.

In the case of a weak gravitational field of point mass, in the arbitrary inhomogeneous plasma (spherically distributed) we have the following formula for the deflection angle:

$$\hat{\alpha}_b = \frac{R_S}{b} + \frac{1}{2}\int\limits_{-\infty}^{\infty}\left(\frac{1}{1-(\omega_e^2/\omega^2)}\,\frac{R_S\, b}{r^3} + \right.$$

$$\left. + \frac{K_e}{\omega^2 - \omega_e^2}\,\frac{b}{r}\,\frac{dN(r)}{dr}\right)dz. \qquad (17)$$

Here ω_e depends on concentration $N(r)$, and $r = \sqrt{b^2 + z^2}$, z is the axis of propagation of the unperturbed straight light ray.

To demonstrate the physical meaning of different terms in (17), we write this expression under condition $1 - n = \omega_e^2/\omega^2 \ll 1$. Carrying out the expansion of terms with the plasma frequency, we obtain:

$$\hat{\alpha}_b = \frac{2R_S}{b} + \frac{1}{2}\frac{R_S\, b}{\omega^2}\int\limits_{-\infty}^{\infty}\frac{\omega_e^2}{r^3}\,dz +$$

$$+\frac{1}{2}\frac{K_e b}{\omega^2}\int\limits_{-\infty}^{\infty}\frac{1}{r}\frac{dN(r)}{dr}dz+ \tag{18}$$

$$\frac{1}{2}\frac{K_e b}{\omega^4}\int\limits_{-\infty}^{\infty}\frac{\omega_e^2}{r}\frac{dN(r)}{dr}dz.$$

The first term is a vacuum gravitational deflection. The second term is an additive correction to the gravitational deflection, due to the presence of the plasma. This term is present in the deflection angle both in the inhomogeneous and in the homogeneous plasma, and depends on the photon frequency. The third term is a non-relativistic deflection due to the plasma inhomogeneity (the refraction). This term depends on the frequency, but it is absent if the plasma is homogeneous. The forth term is a small additive correction to the third term. If we use the approximation $1 - n = \omega_e^2/\omega^2 \ll 1$, and neglect small second and the forth terms, we obtain a separate input of the two effects: the vacuum gravitational deflection, and the refraction deflection in the inhomogeneous plasma. For the arbitrary n one needs to use the expression (17).

Formulae (12) and (17) take into account the gravitation of only point mass (or spherical mass with a size which is less than the impact parameter) and don't take into account gravitation of plasma particles. But the Schwarzschild lens is an idealization, the description of the mass distribution as the point mass is only rarely sufficient for gravitational lens situations, Schneider, Ehlers & Falco (1992). If we want to consider the distribution of gravitating mass (not only point mass) or take into account the mass of the plasma particles, we should use the following formula:

$$\hat{\alpha} = \frac{4GM(b)}{c^2 b} + \frac{2GM(b)\,b}{c^2\omega^2}\int\limits_0^{\infty}\frac{\omega_e^2}{r^3}dz+$$

$$+\frac{K_e b}{\omega^2}\int\limits_0^{\infty}\frac{1}{r}\frac{dN(r)}{dr}dz+\frac{K_e b}{\omega^4}\int\limits_0^{\infty}\frac{\omega_e^2}{r}\frac{dN(r)}{dr}dz=$$

$$= \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 + \hat{\alpha}_4. \tag{19}$$

Here $M(b)$ is the projected mass enclosed by the circle of the radius b. In another words it is the mass inside the cylinder with the radius b. The term $\hat{\alpha}_1$ is the vacuum gravitational deflection, the term $\hat{\alpha}_2$ is the correction to the gravitational deflection, due to the presence of the plasma, the term $\hat{\alpha}_3$ is the refraction deflection due to the inhomogeneity of the plasma, the term $\hat{\alpha}_4$ is a correction to the third term. We are interested mainly in the effects, described by the terms $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$. It is interesting to investigate situations when gravitational effect due to plasma $\hat{\alpha}_2$ (new effect) is comparable with refraction effect $\hat{\alpha}_3$.

We have calculated analytically the terms $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$ for different gravitational lens models. We have considered singular isothermal sphere and non-singular isothermal sphere models. We assume in these cases that the plasma has the similar distribution as the gravitating matter. For the concentration of the plasma we have:

$$N(r) = \frac{\rho(r)}{\kappa m_p}, \tag{20}$$

where m_p is the proton mass, and κ is a non-dimensional coefficient responsible for the dark matter contribution, and is approximately equal $\kappa \simeq 6$. We assume here that $\rho(r)$ is the density of all kind of matter, not only plasma particles. Thus, the plasma frequency is equal to

$$\omega_e^2 = K_e N(r) = \frac{K_e}{\kappa m_p}\rho(r). \tag{21}$$

In the realistic cases $\left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| \ll 1$ in these models.

We also consider self-consistent models of plasma sphere around a black hole and plasma in a galaxy clusters. We assume the gravitating point mass and the gravitating singular isothermal sphere correspondingly, and solve equations of hydrostatic equilibrium to find distribution of the isothermal plasma with temperature T in such gravitational fields, neglecting the mass of plasma. In these models we obtain that

$$\left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| = \frac{2\Re T}{c^2}. \qquad (22)$$

The gravitational correction in plasma may be identified when plasma non-uniformity is not prevailing, what is possible in relativistic plasma, $kT \sim m_e c^2$.

4. Conclusions

1. In the homogeneous plasma gravitational deflection differs from vacuum (Einstein) deflection angle and depends on frequency of the photon. So gravitational lens in plasma acts as a gravitational radiospectrometer.

2. In the observations the spectra of two images may be different in the long wave side due to different plasma properties along the trajectory of the images.

3. The extended image may have different spectra in different parts of the image, with a maximum of the spectrum shifting to the long wave side in the regions with a larger deflection angle.

4. The gravitational effect can be important in the case of a hot gas in the gravitational field of a galaxy cluster.

5. Discussion

J. BEALL: Can you estimate the mass of the planet that produces the spike in the light curve? How large was it ?

G. BISNOVATYI-KOGAN The mass of this planet (OGLE-2005-BLG-390Lb) is estimated as 5.5 Earth masses. For the moment, this is the most distant from us planetary system.

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