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Accretion disc theory since Shakura and Sunyaev

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Abstract. I briefly review the progress of accretion disc theory since the seminal paper of Shakura and Sunyaev.

Key words. Accretion disc - Cataclysmic variables - Dwarf novae

1. Introduction

Discs are a natural occurrence in astrophysical systems whenever they have significant angular momentum. Astronomers had discussed discs of various kinds through the twentieth century. However the paper of Shakura and Sunyaev (1973) transformed the subject, partly because it unified concepts already discussed, and partly through technical innovation.

The most important results of the paper were

(a) the condition for an an accretion disc be *thin*, i.e. to have scale height H much less than disc radius R: Shakura & Sunyaev (1973) showed that the conditions 'thin', 'efficiently cooled' and 'Keplerian' are precisely equivalent: if one of them fails, so do the other two.

(b) it is perfectly possible for an accreting object to be supplied with mass at a rate that ultimately would produce a luminosity above the Eddington limit. Shakura & Sunyaev (1973) suggested that much of the excess would be blown away by radiation pressure at the radius where this luminosity was first reached.
We now know that this is probably what occurs in SS433, and most, if not all, ultraluminous X–ray sources (ULXs) (cf Begelman et al., 2006; Poutanen et al., 2007).

(c) the effective temperature profile of a steady thin disc goes as $T(R) \propto R^{-3/4}$, and this result is *independent* of the mechanism for gas to lose angular momentum and spiral inwards.
This effective temperature profile is now well attested, particularly by observations of CVs. The overall stretched–out blackbody continuum spectrum agrees with it, and more directly, the surface brightness distribution measured during accretion disc eclipses in CVs also agrees: eclipses are broad and shallow at long wavelengths and deep and narrow at short ones.

(d) the mechanism for angular momentum removal (Shakura & Sunyaev (1973) called it 'viscosity') may have a magnetic origin.

(e) in all cases one can parametrize the (kinematic) viscosity as $\nu = \alpha\, c_s H$, where c_s is the local sound speed, and α a quantity of order unity.

This 'alpha prescription' has the great virtue of neatly separating the 'vertical' and

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'horizontal' disc structure. However it is vital to realise that *accretion disc theory is still incomplete*, since we do not know $\nu(\mathbf{x}, t)$. Thus viscosity plays a similar role in accretion disc theory to that played by nuclear burning in stellar evolution theory in the early 20th century.

2. Progress

Just as astronomers were nevertheless able to make some progress with stellar structure theory despite not understanding nuclear burning (cf Eddington's book *The Internal Constitution of the Stars*), theorists and observers have managed to understand how accretion discs behave in some situations. Much of this understanding has come because the disc diffusion equation

$$\frac{\partial\Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left(R^{1/2}\frac{\partial}{\partial r}[\nu\Sigma R^{1/2}]\right) \tag{1}$$

defines a viscous timescale

$$t_{\rm visc} \sim \frac{R^2}{\nu} \tag{2}$$

which can be rewritten using the alpha–prescription as

$$t_{\rm visc} \sim \frac{1}{\alpha}\left(\frac{R}{H}\right)^2 t_{\rm dyn} \tag{3}$$

where $t_{\rm dyn} = (R^3/GM)^{1/2}$ is the local dynamical time.

Thus for example we now know that superhumps result from the presence of the orbital 3:1 resonance within a sufficiently large accretion disc (corresponding to a fairly extreme mass ratio in a CV: (cf Whitehurst & King, 1991; Lubow, 1991, 1992).

Here $t_{\rm visc}$ governs the overall timescale, but variations of it are unimportant for understanding superhumps. Similarly, the possibility of disc instabilities (see the talk by Lasota at this meeting) requires one to imagine only two things: (i) disc structure differs radically if hydrogen is predominantly ionized or not, and (ii) hotter regions have higher viscosity and evolve faster than cool ones. Perhaps suprisingly, the behaviour of the instability in a disc strongly irradiated by the central (X–ray) source is qualitatively much easier to understand (King & Ritter, 1998), since the irradiation traps the disc in the hot state, allowing a pure viscous–timescale decay.

These are particularly simple – pure exponentials – if the disc is small enough that the central source keeps it hot at all radii (the so–called FRED – fast rise, exponential decay systems).

These cases and others give straightforward estimates of α as $\alpha \sim 0.1 - 0.4$. By now one can arrive at some kind of understanding of how almost any pattern of light–curve behaviour can be understood using this picture of disc instability, and possibly allowing for some mild intrinsic variability (e.g. magnetic spots) on the secondary (King & Cannizzo, 1998).

In summary we can say that disc 'theory' works quite well provided that we *assume* these values of α.

However this apparent success comes at a double price. First, it is entirely ad hoc – it works (rather like the Old Quantum Theory) because of a series of fudges and empirical rules. Second, and more seriously, it means that we are entirely unable to predict, or confirm or deny, *global* changes in disc structure, such as whether thin disc accretion can make a transition to advection–dominated (ADAF) flow, or just *how* an accretion disc creates and powers a jet at its centre. This is analogous to the inability of pre–nuclear stellar structure theory to predict or explain supernovae.

3. The answer?

In searching for the true mechanism for angular momentum transport we need to remember the distinctive feature of accretion discs, that they simultaneously obey

$$\frac{\partial}{\partial R}(R^2\Omega) > 0, \text{ and } \frac{\partial\Omega}{\partial R} < 0 \tag{4}$$

where Ω is the local angular velocity (the Kepler value $(GM/R^3)^{1/2}$ in a thin disc). That is, angular momentum increases outwards, but angular velocity decreases outwards. The first property tell us that discs are stable against axisymmetric perturbations (Rayleigh criterion), so removing a large number of candidate mechanisms. Indeed, most purely hydrodynamical mechanisms are sensitive to the gradient of angular momentum rather than velocity, and so would if anything transport angular momentum inwards.

The dragging of a magnetic fieldline anchored in an accretion disc on the other hand *is* sensitive to the angular velocity gradient, and does offer a promising candidate (Balbus & Hawley 1991) for the transport mechanism. But although this is clear, actually calculating the full effect of this process is a formidable challenge. In principle one has to solve the full disc structure self–consistently, describing gas motions in full 3D time–dependent MHD. Most theoretical effort so far has gone into trying us numerical simulations to quantify the viscous transport (naively, estimate α) in the so–called 'shearing box' approximation. Here one considers a corotating Cartesian box, plus tidal gravity and Coriolis terms. This is a much more tractable problem, but has inevitable limitations. Most obviously, the scale of the box is only $\sim H$, so the simulations are only sensitive to high wavenumbers $\sim R/H$, and hence small–scale magnetic fields.

The results of this procedure are mixed (see King, Pringle & Livio, 2007 for a review).

Fully–ionized shearing–box simulations tend to give rather small values $\alpha \sim 0.02$, unless a vertical magnetic field is imposed from the start. Worse, there is some indication that the value of α is resolution–dependent (Fromang & Papaloizou, (2007) in the sense that α decreases as the numerical resolution of a simulation is increased. So although MHD effects are probably the basis of accretion disc viscosity, current simulations have not convincingly shown this, still less that the effect is large enough to account for observations. Evidently this problem will require global disc simulations, and so even more powerful computers.

4. Conclusions

Our current ad hoc picture of accretion discs is based on the semi–empirical ideas of Shakura and Sunyaev (1973). It works reasonably well in a number of areas, provided that we assume $\alpha \sim 0.1 - 0.4$. However it is still ad hoc, and is unable to predict or confirm global changes of disc structure. The real basis of accretion disc viscosity is probably magnetic, as suggested by Balbus and Hawley in 1991. Attempts to demonstrate this with shearing–box simulations produce viscosities which are too weak (i.e. $\alpha \lesssim 0.02$) compared with observed constraints, and may even be resolution–dependent. Balbus and Hawley's paper appeared less than 20 years after Shakura & Sunyaev (1973), but 20 years further on the huge complexity of the viscosity problem has markedly slowed practical progress. We are still a long way from a theory of accretion discs with real predictive power.

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