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Putting Einstein to test

Astrometric experiments in fundamental physics

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Abstract. In classical Astronomy, the main goal of Astrometry was the determination of an inertial frame of reference, which, in modern language, can be intended as an experiment of fundamental physics. In relativistic physics the concept of an absolute inertial frame of reference cannot hold anymore, nonetheless Astrometry retains its role as an experimental counterpart of fundamental physics. Modern high-precision astrometry, in fact, must be formulated on the theoretical background of relativistic physics and of a relativistic theory of measure. On the other side, it can be used to put to test several topics involved in the quest among the different theories of gravity. This talk will try to give a brief overview on the connections between some of these topics and their formulation as an astrometric experiment.

1. Introduction

Although General Relativity (GR) is still the most favoured theory providing a detailed description of the Gravitational interaction beyond the Newtonian limit, several other alternatives have been proposed since GR was developed. Examples of such theories are those where the field equations contain not only the metric tensor of GR, but also a scalar field coupled with the metric itself (Will , 2006), or the so-called fourth-order theories of gravity, where the scalar of curvature R in the field equations is replaced by a more complex function of this quantity, $f(R)$ (Capozziello and Faraoni, 2011).

The most popular mathematical tools used to discriminate among the possible theories is the so-called *Parametrized Post-Newtonian framework* which enables the comparison of several theories through the estimation of the value of a limited number of parameters. The most important parameters for experiments based on astrometric measurements are γ and β since they are connected with the classical astrometric phenomena of the light deflection and of the excess of perihelion precession in the orbits of massive objects.

Besides its immediate implication for the fundamental physics problem of characterizing the best gravity theory, a precise estimation of these parameters has important consequences on the interpretation of observational evidences at different scales in space and time up to cosmological scales. A precise estimation of these two parameters, has important *theoretical and observational implications*. It could help to *fulfill theoretical needs* because they are the phenomenological "trace" of a scalar field

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coupled with gravity of scalar-tensor theories which is related to:

- theories fully compatible with the Mach principle (Brans and Dicke, 1961);
- cosmological scenarios with inflationary stage (Damour and Nordtvedt , 1993);
- theories aiming to provide a formulation of a quantum theory of gravity (Damour et al. , 2002).

For example, there are formulations of scalar-tensor theories in which the scalar field evolves with time toward a theory close to GR leaving little relic deviations at present times (Damour and Esposito-Farèse , 1992). Such deviations from GR today range from 10^{-5} to a few times 10^{-7} for $|\gamma-1|$, depending on the cosmological model (Damour and Nordtvedt , 1993).

On the front of the $f(R)$ theories, instead, it has been argued that the current estimations of the γ and β PPN parameters are not sufficient to provide serious constraints on such theories, which claim of being able to explain *observational evidences* about several astrophysical and cosmological problems without any need for Dark Matter (DM) or Dark Energy (DE) like, e.g. Capozziello et al. (2009):

- DE dynamics (acceleration of cosmological expansion);
- DM dynamics (galactic rotation curves, galaxy cluster masses);
- observational data from gravitational lensing;
- Tully-Fisher relation.

Again, it seems that the desired experimental accuracy can be set at level of 10^{-7} to 10^{-8} for $|\gamma - 1|$, and from 10^{-5} for $|\beta - 1|$ (Capozziello et al., 2006).

Among the other useful tools with a possible importance for astrometric-based experiments, we can cite the *Mansouri-Sexl formalism* (Mattingly , 2005) which can in principle provide a way to test the foundations of Special Relativity which are connected to different approaches toward the quantization of gravity.

2. Basics of measurement theory in a relativistic astrometric framework

In order to understand why Astrometry can be used to set limits to the PPN parameters and therefore to put to test the different theories of gravity, it has to be shown how these parameters enter the astrometric observable. The most basic measure in Astrometry, ideally, is the angle ψ_{12} between two observing directions, which in the usual Euclidean geometry reads

$$\cos\psi_{12} = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|}, \tag{1}$$

and can be represented as the projection on a unit sphere of the two vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ connecting the observer and the objects P and Q.

In order to give a correct interpretation of the experimental results, however, the measurements must be written in a proper relativistic way, i.e. following the prescription of the *theory of measurements*. The details of this formalism are out of scope in this paper, and are fully developed in de Felice and Bini (2010). We therefore will give here only a brief overview of the main concepts needed to follow the present exposition.

The relativistic counterpart of Eq. (1) can be written as

$$\cos\psi_{12} = \frac{\mathcal{T}_{\alpha\beta}k_1^\alpha k_2^\beta}{\sqrt{\mathcal{T}_{\alpha\beta}k_1^\alpha k_1^\beta}\,\sqrt{\mathcal{T}_{\alpha\beta}k_2^\alpha k_2^\beta}} \tag{2}$$

where, k_1^α and k_2^α are the two observing directions (de Felice and Clarke, 1990), and $\mathcal{T}_{\alpha\beta}$ is an operator that depends on the space-time metric $g_{\alpha\beta}$ and on the four-velocity of the observer u^α. It has to be noticed, however, that the expression “observing direction” has not to be intended as the result of a measure. The observable is the angle at the left-hand-side of the equation, while the four-vectors $k_{1,2}^\alpha$ have the mathematical meaning of the tangents to the path of the incoming light rays connecting the positions of the observed objects to the observer, i.e. of the null geodesics of the observed photons. In other words, if s is the parameter of the null geodesic x^α, then

$$k^\alpha = \frac{\mathrm{d}x^\alpha}{\mathrm{d}s}$$

and this four-vector can be obtained by solving the geodesic equation (i.e. the relativistic equation of motion)

$$\frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}s^2} + \Gamma^\alpha_{\beta\gamma} \frac{\mathrm{d}x^\beta}{\mathrm{d}s} \frac{\mathrm{d}x^\gamma}{\mathrm{d}s} = 0. \tag{3}$$

This formula has to be expressed as a function of the desired astrometric unknowns, namely the positions and proper motions of the observed objects. Moreover, a relativistic formulation of the astrometric measurements has to properly take into account the geometry of the space-time, and this is the way the PPN parameters γ and β enter the observation equations.

3. Other kind of astrometric experiments. Beyond the PPN framework

Other interesting phenomena which can be used to test the gravity theories at a fundamental level are the higher-order quadrupole contribution to the light deflection, and the possible violations of the Local Lorentz Invariance (LLI). Both can be put to test with convenient astrometric experiments.

The first one is foreseen by GR and other theories of gravity when the light is deflected by perturbing bodies with non-spherically symmetric distributions of the mass. The coefficients of $g_{\alpha\beta}$, in fact, in this case does not depend only on the mass, but also on the higher order multipoles of the gravity field of the perturbing body. The deflection can then be described as a vectorial quantity with two components $\mathbf{n}$ and $\mathbf{m}$

$$\Delta\psi = \Delta\psi_1 \mathbf{n} + \Delta\psi_2 \mathbf{m}$$

with respect to a reference triad $(\mathbf{n}, \mathbf{m}, \mathbf{z})$ where the $\mathbf{z}$-axis is orthogonal to the celestial sphere.

The two components can be modelled as functions of a parameter ε whose value is 1 in GR like in the following formulae (Crosta and Mignard, 2006)

$$\begin{aligned} \Delta\psi_1 &= \frac{2\,(1+\gamma)\,M}{b}\left[1 + \varepsilon J_2 \frac{R^2}{b^2}\right. \\ &\qquad \left.\left(1 - 2\,(\mathbf{n}\cdot\mathbf{z})^2 - (\mathbf{t}\cdot\mathbf{z})^2\right)\right] \qquad (4) \\ \Delta\psi_2 &= \frac{4\,(1+\gamma)\,M\varepsilon J_2 R^2 \gamma}{b^3}\,(\mathbf{m}\cdot\mathbf{z})\,(\mathbf{n}\cdot\mathbf{z}) \qquad (5) \end{aligned}$$

where M is the mass of the perturbing body, J_2 is the quadrupole component of its gravitational field, and b the impact parameter, i.e. the distance of maximum approach of the light path to the perturbing body.

This asymmetric perturbation on the light path induces specific patterns in the nearby light deflection which have never been measured up to now because of the smallness of this effect.

Tests of possible violations of the LLI are motivated by several theoretical models encompassing a large number of different subjects, from quantum gravity to varying speed of light cosmologies. A complete review of the tests and of the motivations linked to the LLI is out of scope here, and can be found in Mattingly (2005). Here we will limit ourselves to cite the Robertson-Mansouri-Sexl (RMS) formalism for its possible application to astrometry. A violation of the LLI, in fact, will show itself as a breaking of the Lorentz transformations. The RMS formalism is a way to describe this hypothetical breaking in a kinematical way.

In analogy to the PPN formalism, the RMS framework is developed under the assumption that $v \ll c$, and can be expressed as a generalization of the Lorentz transformations

$$\begin{aligned} T &= \frac{(t - \boldsymbol{\epsilon}\cdot\mathbf{x})}{a} \\ \mathbf{X} &= \frac{\mathbf{x}}{d} - \left(\frac{1}{d} - \frac{1}{b}\right)\frac{\mathbf{v}(\mathbf{v}\cdot\mathbf{x})}{v^2} + \frac{\mathbf{v}}{a}t \end{aligned} \tag{6}$$

depending on a set of arbitrary parameters $(a, b, d, \boldsymbol{\epsilon})$. The potential impact of this formalism on astrometric measurements comes from the fact that those LLI violations depending on $f = d/b$, show themselves as an aberration effect which therefore puts to test the same properties of the Michelson-Morley experiment. It is probable that violations depending on such aberration effect are out of reach for the present planned space experiments like Gaia (Klioner, 2008) but possible applications to new and more sensitive experiments has not been investigated yet.

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