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A gentle introduction to modified gravity

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Abstract. In the last ten years, models of modified gravity have flourished. Most of them are motivated by the late time acceleration of the Universe and the possibility of modifying the dynamics of the Universe compared to the Λ-CDM paradigm on large scales. It turns out that all such known models involve at least one scalar field which couples to matter. This leads to gravitational problems in the solar system which can be evaded thanks to one of the three known screening mechanisms: chameleon, Damour-Polyakov and Vainshtein, and also the possibility of seeing temporal and spatial variations of constants.

1. Introduction

The acceleration of the expansion of the Universe could be due to a lack of understanding of gravity on large scales (Khoury 2010). In the last ten years, models of modified gravity have appeared and all involve a scalar degree of freedom. In some sense, one can see the origin of this scalar as emanating from the simple fact that a massive graviton has five polarisations, one being scalar. One main feature of these models is that they act on large scales implying that the range of the interaction mediated by the scalar field must be much larger than the solar system. This would be ruled out by solar system tests of gravity such as the observations by the Cassini probe (Bertotti et al. 2003) if screening mechanisms did not manifest themselves in dense environments.

These mechanisms require the introduction of non-linearities in the models and there seems to be essentially three ways of doing so. The first two follow the original chameleon model (Khoury and Weltman 2003; Khoury and weltman 2004; Brax et al. 2004) and use the density dependence of the effective potential describing the behaviour of the scalar field in the presence of matter. In this description, two ingredients can be changed: the bare scalar potential or the coupling of the scalar to matter. Chameleon models have a non linear bare potential and a linear coupling to matter while models subject to the Damour-Polyakov mechanism (Damour and Polyakov 1994; Pietroni 2005; Olive and Pospelov 2007; Brax et al. 2010; Hinterbichler and Khoury 2010) have a non-linear coupling satisfying the least coupling principle, i.e. the existence of a field value for which the coupling to matter vanishes. These two approaches can be totally unified in the same description (Brax et al. 2012). One can also introduce non-linearities in the kinetic terms of the scalar field. For these models, the Vainshtein mechanism (Vainshtein 1972) protects the local properties of gravity. We will not deal with the latter and focus on the first two mechanisms only. We will explain how their dynamics lead to the possible existence of variation of constants.

2. Modified gravity

2.1. Effective description

Gravity as described by General Relativity can be modified by the introduction of at least one scalar field. We will analyse the modification of gravity in the Einstein frame where the Einstein-Hilbert action is not altered

$$S_{EH} = \int d^4x \sqrt{-g}\frac{R}{2\kappa_4^2}, \tag{1}$$

R is the Ricci scalar of the Einstein frame metric $g_{\mu\nu}$. We have identified $\kappa_4^2 = 8\pi G_N = m_{\rm Pl}^{-2}$ where $m_{\rm Pl}$ is the reduced Planck scale. The scalar part of the action involves both the scalar field and matter fields with no specific form of preferred couplings. In general, such a field theory is very complex to analyse and we shall assume that in a given environment described by macroscopic bodies with non-relativistic matter the theory admits a vacuum configuration ϕ_0 which depends on the distribution of matter. Let us expand the scalar Lagrangian of the theory up to second order in the small variation $\delta\phi$ of the scalar field around the background value ϕ_0 and retain the following relevant terms

$$\mathcal{L} \supset -\frac{Z(\phi_0)}{2}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{m_{\rm Pl}}\delta\phi\delta T\ . \tag{2}$$

Up to second order, the scalar field couples to the variation of the trace of the matter energy momentum tensor δT via the coupling constant $\beta(\phi_0)$. The wave function normalisation $Z(\phi_0)$ appears in models with higher order derivatives. The mass of the scalar field $m(\phi_0)$ depends on the environment too. Here, gravity is modified in as much as the coupling of ϕ to matter implies a modification of the geodesics compared to General Relativity. They depend now on the full Newtonian potential

$$\Phi = \Phi_N + \beta(\phi_0)\frac{\phi}{m_{\rm Pl}} \tag{3}$$

where Φ_N is the Newtonian potential satisfying the Poisson equation.

Let us consider a test particle with no self-gravity evolving in this environment. We assume that this test particle is subject to the presence of a large and point-like body at the origin of coordinates. At the linear level and in the quasi static approximation (where gradients are much larger than time derivatives), the field equation is simply

$$Z(\phi_0)\Delta\delta\phi - m_0^2\delta\phi = -\beta(\phi_0)\frac{\delta T}{m_{\rm Pl}} \tag{4}$$

where $\delta T = -M\delta^{(3)}(r)$ whose solution is given by

$$\delta\phi = -\frac{\beta(\phi_0)}{Z(\phi_0)}\frac{M}{4\pi m_{\rm Pl}^2 r}e^{-m(\phi_0)r/Z^{1/2}(\phi_0)}. \tag{5}$$

For distances much less than the Compton wavelength $\lambda = \frac{Z^{1/2}(\phi_0)}{m(\phi_0)}$, the total Newtonian potential is given by

$$\Phi = (1 + 2\frac{\beta^2(\phi_0)}{Z(\phi_0)})\Phi_N \tag{6}$$

The scalar force is screened by the Vainshtein mechanism (Vainshtein 1972) when $Z(\phi_0)$ is large enough that the coupling of the normalised field $\beta(\phi_0)/Z^{1/2}(\phi_0)$ is small enough. The chameleon mechanism (Khoury and Weltman 2003; Khoury and weltman 2004) occurs when the mass $m(\phi_0)$ is large enough to suppress the range of the scalar force in dense environments. Finally, the Damour-Polyakov screenings is such that $\beta(\phi_0)$ itself is small (Damour and Polyakov 1994).

2.2. The importance of non-linearities

At the non-linear level all the models that we shall consider are described by scalar-tensor theories defined in the Einstein frame by a Lagrangian

$$S = \int d^4x \sqrt{-g^E}(\frac{R_E}{16\pi G_N} - F(\partial,\phi) - V(\phi)) + S_m(\psi, A^2(\phi)g^E_{\mu\nu}) \tag{7}$$

where $A(\phi)$ is an arbitrary function and F represents the generalised kinetic terms which can be of higher order. The coupling to matter that we have already introduced is simply given by

$$\beta_\phi = m_{\rm Pl}\frac{d\ln A(\phi)}{d\phi}. \tag{8}$$

A theory with a linear coupling β is such that $A(\phi)$ is an exponential function. The most important features of models with canonical kinetic terms such as chameleons, dilatons and symmetrons is that the scalar field dynamics are determined by an effective potential which takes into account the present of the conserved matter density ρ of the environment:

$$V_{\rm eff}(\phi) = V(\phi) + (A(\phi) - 1)\rho. \tag{9}$$

With a decreasing $V(\phi)$ and an increasing $A(\phi)$, the effective potential acquires a matter dependent minimum $\phi_{\rm min}(\rho)$ where the mass is also matter dependent $m(\rho)$. These properties are at the heart of the chameleon and the Damour-Polyakov mechanisms.

We can now describe the way screening of the scalar interaction appears in dense environments such as the solar system. In screened massive bodies, the field value departs very little from the minimum of the effective potential deep inside the body. Denoting by ϕ_c this value and by ϕ_∞ the value outside and far away from the body, an approximate solution of the Klein-Gordon equation in the spherical case is simply

$$\phi = \phi_c, \ r \le R$$

$$\phi = \phi_\infty + (\phi_c - \phi_\infty)\frac{R}{r}, \ r \ge R,$$

which describes accurately the outside solution in the screened case (Brax et al. 2012).One can write

$$\phi(r) = \phi_\infty - Qm_{\rm Pl}\frac{G_N M}{r} \tag{10}$$

where M is the mass of the dense object and R its radius. We have defined the scalar charge

$$Q = \frac{\phi_\infty - \phi_c}{m_{\rm Pl}\Phi_N}. \tag{11}$$

where Φ_N is here the value of Newton's potential at the surface of the body $\Phi_N = \frac{G_N M}{R}$. The scalar charge depends on the environment via ϕ_∞ and on the properties of the body via ϕ_c and ϕ_N. Comparing the solution for a point-like source

$$\phi = \phi_\infty - \beta_\infty m_{\rm Pl}\frac{G_N M}{r} \tag{12}$$

we immediately find that the screening criterion (Khoury and weltman 2004) is

$$Q \ll \beta_\infty \tag{13}$$

There are essentially two types of screening: an object is *self-screened* when its Newtonian potential is large enough, it is *environmentally screened* when both β_∞ and ϕ_∞ conspire to satisfy the screening criterion.

2.3. Tomography

Scalar-tensor theories whose effective potential $V_{\rm eff}(\phi)$ admits a density dependent and stable minimum $\phi(\rho)$ can all be described parametrically from the sole knowledge of the mass function $m(\rho)$ and the coupling $\beta(\rho)$ at the minimum of the potential as long as they are canonically normalised, i.e. excluding the Vainshtein mechanism. Identifying the mass as the second derivative

$$m^2(\rho) = \frac{d^2 V_{\rm eff}}{d\phi^2}|_{\phi=\phi(\rho)} \tag{14}$$

and the coupling

$$\beta(\rho) = m_{\rm Pl}\frac{d\ln A}{d\phi}|_{\phi=\phi(\rho)}. \tag{15}$$

we have the integral

$$\frac{\phi(\rho)-\phi_c}{m_{\rm Pl}} = \frac{1}{m_{\rm Pl}^2}\int_\rho^{\rho_c} d\rho\frac{\beta(\rho)A(\rho)}{m^2(\rho)} \tag{16}$$

where ϕ_c and ρ_c are taken to be the minimum value and the density inside a dense body such as the earth. Similarly the potential can be obtained as

$$V(\rho) - V_c = -\int_\rho^{\rho_c} d\rho\frac{\beta^2(\rho)A^2(\rho)}{m^2(\rho)m_{\rm Pl}^2}\rho \tag{17}$$

where V_c is also the potential value for the density ρ_c. For most models, in the appropriate density range from a few $g/{\rm cm}^3$ to cosmological densities, the function $A(\rho)$ is essentially constant and equal to one.

It is often simpler to define the functions $m(\rho)$ and $\beta(\rho)$ using the time evolution of the matter density of the Universe

$$\rho(a) = \frac{\rho_0}{a^3} \tag{18}$$

where a is the scale factor whose value now is $a_0 = 1$. For instance the power law models with

$$m(a) = m_0 a^{-r},\ \beta(\rho) = \beta \tag{19}$$

where $r > 3$ correspond to chameleon models with

$$V(\phi) = V_0 + \frac{\Lambda^{n+4}}{\phi^n} \tag{20}$$

and $n = 2(r-3)/(2r-3)$.

The tomographic mapping is particularly useful to express the screening condition for scalar-tensor models

$$\frac{1}{m_{\rm Pl}^2}\int_{\rho_\infty}^{\rho_c} d\rho\frac{\beta(\rho)A(\rho)}{m^2(\rho)} \ll \beta_\infty\Phi_N \tag{21}$$

where ρ_∞ is the density far away from the object. Given $\beta(\rho)$ and $m(\rho)$, this is very easily implemented. as can be seen in the screening of the Milky Way itself. The Milky Way is screened when

$$\frac{3}{m_{\rm Pl}^2}\int_{a_G}^{1} da\frac{\beta(a)\rho(a)}{am^2(a)} \le \beta_0 10^{-6} \tag{22}$$

where $\phi_N \sim 10^{-6}$ for the Milky Way we have assumed that the Milky Way belongs to an unscreened cluster of galaxies. The screening of the Milky Way implies that $m_0/H_0 \gtrsim 10^3$ when $\beta(a)$ varies little between a_G and 1 (Brax et al. 2011; Wang et al. 2012).

3. Variation of constants

In the cosmological context, screened models have two fundamental properties which lead to the variation of constants. The first one is that the field minimum varies with time as the matter density evolves. This tracking leads to a temporal variation of constants. The field is also locally stuck at the minimum of the effective potentials in structures such as galaxy clusters and galaxies where the screening mechanism may be at play. Then depending on the local environment of the absorbing systems where atomic transitions take place compared to the earth, a spatial variation could appear which would be interpreted as a variation of constants between a redshift $z = 0$ and the redshift of the absorbing system. In the simple models that we have presented, only masses vary as the fine structure constant appears in front of the conformally invariant operator F^2 in the electromagnetic Lagrangian.

The variation of masses in the Einstein frame frame follows from

$$m_\psi = A(\phi) m_\psi^{(0)} \tag{23}$$

where $m_\psi^{(0)}$ is the mass in the Jordan frame where all masses are constant. In fact, in the Jordan frame the Planck mass varies and only the variation of the ratio $m_\psi / m_{\rm Pl}$ makes sense. We will refer to the variation of masses as the variation with respect to the Planck scale taken as an immutable ruler. The time evolution of the minimum is given by

$$\dot\phi = 9\beta\Omega_m \frac{H^2}{m^2} H m_{\rm Pl} \tag{24}$$

implying that the cosmological evolution of particle masses (in the cosmological vacuum) is given by

$$\frac{\dot m_\psi}{m_\psi} = 9\beta^2 \frac{H^2}{m^2} H \tag{25}$$

where the coupling β and the mass m at the minimum are time dependent. For screened models where $m_0/H_0 \gtrsim 10^3$ now, this variation in the recent past is only a fraction of the Hubble rate. One caveat is that the time variation calculated in this way would require to observe atomic transition in the cosmological vacuum. When atoms appear to be within bound structures such as galaxies or galaxy clusters, one must take into account the environment and assess whether the atomic transitions occur in screened regions or not. When the environment is screened the local value of the scalar field is the one at the minimum of the effective potential due to the local distribution of masses and not the cosmological one. In this case, one can interpret a difference between measurements on earth and at a redshift z as due to the different values of the scalar field in the two environments.

In this case,the variation of the masses is given

$$\frac{\Delta m_\psi}{m_\psi} = \beta_0 \frac{\Delta\phi}{m_{\rm Pl}} \tag{26}$$

where β_0 is the coupling to matter now and we consider spatial variations corresponding to far away systems emitting light in the recent past of the Universe. This can be applied to the electron to proton mass ratio

$$\frac{\Delta\mu}{\mu} = \beta \frac{\Delta\phi}{m_{\rm Pl}} \tag{27}$$

as the QCD scale is scalar field independent and only the electron mass is scalar-dependent. For probes in the local galactic environment, we can use the tests of the equivalence principle in the solar system performed by the Lunar Ranging experiment (Williams et al. 2012) to deduce that (Khoury and weltman 2004)

$$\frac{\Delta\phi}{m_{\rm Pl}} \le \beta_0 Q_\oplus \Phi_\oplus \tag{28}$$

where local tests imply that $Q_\oplus \le 10^{-7}$ and $\Phi_\oplus \sim 10^{-9}$ This leads to the bound

$$\frac{\Delta\mu}{\mu} \le \beta_0^2 Q_\oplus \Phi_\oplus \lesssim 10^{-16}\beta_0^2 \tag{29}$$

For models with $\beta_0 \sim 1$ this is 8 orders of magnitude lower than the present experimental bound in the Milky Way (Rahmani et al.

2013). For absorbers outside the Milky Way the tomographic mapping allows one to write

$$\frac{\Delta\mu}{\mu} = 9\beta_0 \int_{a_0}^{a_1} \frac{\beta(a)\Omega_m(a)H^2(a)}{am^2(a)} da \qquad (30)$$

where a_0 and a_1 are the scale factors corresponding to the local environments on earth and far away. On earth and taking the values of masses in the atmosphere where the density is large we can take $a_0 \sim 10^{-8}$. If the absorbers are in an unscreened environment in the cosmological vacuum we have $a_1 = 1/(1+z_1)$ where z_1 is the redshift of the absorbing system. If the absorber is in a screened regions with density similar to the Milky way then $a_1 \sim 10^{-2}$. In the unscreened case, the variation is at most of the order 10^{-6} and much lower in the screened case.

4. Conclusion

Screened models of modified gravity subject to the chameleon or the Damour-Polyakov mechanisms are such that particle masses, and in particular the electron to proton mass ratio, evolves in time in the cosmological vacuum. Moreover, the particle masses in different bound system such as the Milky Way and far away galaxies would be different depending on whether the scalar field responsible for the modification of gravity is screened in the different local environments. This would appear as a spatial variation of constants which would then be interpreted as a variation as a function of the redshift of the absorbing systems where atomic transitions take place.

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