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Equivalence principle in Chameleon models

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Abstract. Most theories that predict time and/or space variation of fundamental constants also predict violations of the Weak Equivalence Principle (WEP). Khoury and Weltmann proposed the chameleon model in 2004 and claimed that this model avoids experimental bounds on WEP. We present a contrasting view based on an approximate calculation of the two body problem for the chameleon field and show that the force depends on the test body composition. Furthermore, we compare the prediction of the force on a test body with Eötvös type experiments and find that the chameleon field effect cannot account for current bounds.

Key words. equivalence principle – gravity: theories

1. Introduction

Most theories that predict variation of fundamental constants, also predict violations of the Weak Equivalence Principle (WEP) Bekenstein (1982); Barrow et al. (2002); Olive & Pospelov (2002); Damour & Polyakov (1994); Palma et al. (2003). The reason for this is that the mass of a body is made of many contributions related to various interaction energies (strong, weak, electromagnetic). Therefore any theory in which the local coupling constants become effectively spatially dependent through their direct dependence on an light scalar field will entail some non-universality in the free-fall acceleration of bodies embedded in an external gravitational field. On the other hand, WEP is strongly constrained by Eötvös type experiments; latest results give $\frac{\Delta a}{a} \simeq 10^{-14}$. However, some schemes claim to be able to avoid this problem based on proposals such as the chameleon models and the Dilaton-Matter-gravity model with strong coupling Damour et al. (2002). Chameleon models where introduced by Khoury & Weltman (2004) and further developments were performed by several authors Brax et al. (2004); Mota & Shaw (2007). Khoury and Weltman, 2004 have shown that the parameters of the chameleon model were constrained by experi-

Send offprint requests to: L. Kraiselburd mental bounds of the WEP. On the other hand, Mota & Shaw, 2007 claim that while the linear and quasi-linear solution seems to predict violation of WEP, the non-linear solution avoids that prediction at the particle level. According to these authors, the reason for this lies in that the non-linear effects become relevant only on a small region near the body's surface which has been denominated "*the thin shell*". Based on such analysis, it has been argued that the chameleon field does not depend on the composition of the falling body or in general on the interaction between the matter and chameleon fields. In this paper, we perform an alternative calculation of the chameleon mediated force on a free falling body exerted by a large body like a mountain or the Earth. First, we propose an approximate solution of the two body problem and show that the force on a test body depends on its composition. Furthermore we compare the prediction for the differential acceleration between two test objects of different composition, subject to the Earth's or Suns attraction, with the corresponding observational bounds extracted from Eötvös type experiments.

2. The Model

Let us first briefly review the chameleon model. The theory is characterized by the general action:

$$\begin{aligned} S &= \int d^4x\,\sqrt{-g}\left[\frac{M_{pl}}{2}R - (\partial\Phi)^2 - V(\Phi)\right] \\ &\quad - \int d^4x L_m\left(\Psi_m^{(i)}, g_{\mu\nu}^{(i)}\right) \end{aligned} \tag{1}$$

where each matter field $\Psi^{(i)}$ couples to a metric $g_{\mu\nu}^{(i)}$ related to the Einstein-frame metric $g_{\mu\nu}$ by a conformal factor: $g_{\mu\nu}^{(i)} = \exp\frac{2\beta_i\Phi}{M_{pl}}g_{\mu\nu}$, M_{pl} is the Planck mass and the potential is assumed to be $V(\Phi) = M^{4+n}\Phi^{-n}$ where M is a constant. In order to exhibit the chameleon effect we shall assume $\beta_i = \beta$, although it is precisely the fact that each species of particle couples differently to the chameleon field the reason behind the potential violation of the WEP, and so the *thin shell effect* has been considered as the crucial mechanism required to suppress such violations. The dynamics of the chameleon depends crucially on its effective potential according to the following equation:

$$\begin{aligned} \Box\Phi &= \frac{\partial V_{eff}}{\partial\Phi} \\ V_{eff} &= V(\Phi) - T\exp(\frac{\beta\Phi}{M_{pl}}) \end{aligned} \tag{2}$$

where T is the trace of the energy-momentum tensor of the matter occupying the region under consideration.

In order to compute the force on a test body that is free-falling in the presence of a larger body, we have to solve the above equation for the case of two bodies. In this paper, we present an approximate solution for the chameleon field in the presence of two spherical bodies. We expand the most general solution in terms of complete sets of solutions in three regions:i) Inside the large body, ii) Inside the test body and iii) Outside both bodies. For region i) and iii) we keep the dominant term: the one-body solution. Inside the free falling body we seek the most general solution for the two body problem, and impose continuity at the border.

For the benefit of the reader we reproduce the solution of Eq. (2) for a spherically-symmetric body of radius R and density ρ_{in} immersed in an external medium of density ρ_{out} (for details see Khoury & Weltman 2004) being $T_{\text{in(out)}} = -\rho_{\text{in(out)}} + 3P_{\text{in(out)}}$. In this case we must consider 2 regions and $3P \to 0$:

$$\rho = \begin{cases} \rho_{\text{in}} & r \le R \\ \rho_{\text{out}} & r > R \end{cases}$$

The next step is to make suitable expansions of the effective potential about the corresponding minimum both for the outside and inside regions:

$$\begin{aligned} V_{eff}^{\text{in,out}}(\Phi) &\simeq V_{eff}^{\text{in,out}}(\Phi_{\text{min}}^{\text{in,out}}) \\ &\quad + \frac{1}{2}\partial_{\Phi\Phi}V_{eff}^{\text{in,out}}(\Phi_{\text{min}}^{\text{in,out}})(\Phi - \Phi_{\text{min}}^{\text{in,out}}) \end{aligned}$$

where the superscript in(out) refers to inside (outside) of the large body. We define:

$$m_{eff}^{2\text{in,out}}(\Phi_{\min}^{\text{in,out}}, \beta, T_{\text{in,out}}) = \partial_{\Phi\Phi} V_{eff}^{\text{in,out}}(\Phi_{\min}^{\text{in,out}})$$

Thus we are lead to solve the following equations:

$$\frac{1}{r}\partial r(r^2 \partial r \Phi^{\text{in}}) = m_{eff}^{2\text{in}}(\Phi^{\text{in}} - \Phi_{\min}^{\text{in}})$$
$$\frac{1}{r}\partial r(r^2 \partial r \Phi^{\text{out}}) = m_{eff}^{2\text{out}}(\Phi^{\text{out}} - \Phi_{\min}^{\text{out}})$$

with the border conditions:

$$\begin{aligned}
\Phi^{\text{in}}(r=0) &= \Phi_0 \\
\partial r \Phi^{\text{in}}(r=0) &= 0 \\
\Phi^{\text{out}}(r \to \infty) &= \Phi_{\min}^{\text{out}} = \Phi_\infty
\end{aligned}$$

and the condition for both solutions (in, out) to match is:

$$\begin{aligned}
\Phi^{\text{in}}(r=R) &= \Phi^{\text{out}}(r=R) \\
\partial r \Phi^{\text{in}}(r=R) &= \partial r \Phi^{\text{out}}(r=R)
\end{aligned}$$

The solution for the one body problem for $r \leq R$ is:

$$\Phi^{\text{in}}(r) = \frac{(\Phi_0 - \Phi_c)\sinh(m_{eff}^{\text{in}} r)}{m_{eff}^{\text{in}} r} + \Phi_c$$

where $\Phi_c = \Phi_{\min}^{\text{in}}$ and

$$\Phi_0 = \Phi_c + \frac{(\Phi_\infty - \Phi_c)\left[1 + m_{eff}^{\text{out}} R\right]}{\frac{m_{eff}^{\text{out}} R}{x}\sinh(x) + \cosh(x)}$$

with $x = m_{eff}^{in} R$. For $r \geq R$ we get:

$$\Phi^{\text{out}}(r) = C\frac{\exp(-m_{eff}^{\text{out}} r)}{r} + \Phi_\infty$$

where

$$C = \frac{R(\Phi_c - \Phi_\infty)\left[\cosh(x) - \frac{\sinh(x)}{x}\right]\exp(m_{eff}^{out} R)}{\frac{m_{eff}^{\text{out}} R}{x}\sinh(x) + \cosh(x)}$$

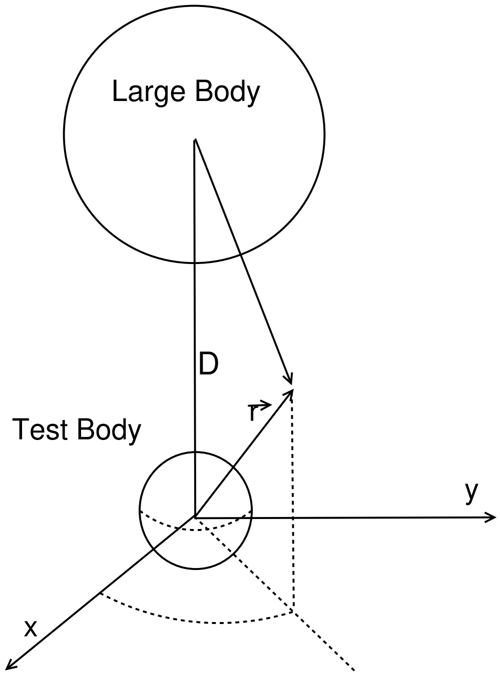

Fig. 1. Two body problem.

Now we take the thin-shell approximation (i.e. $1 \ll m_{eff}^{in} R$) and for simplicity assume $m_{eff}^{\text{out}} R \ll 1$, and obtain approximate expressions for the field inside and outside the large body respectively:

$$\Phi^{\text{in}}(r) \approx \frac{2(\Phi_\infty - \Phi_c)\exp[-R m_{eff}^{\text{in}}]\sinh(m_{eff}^{\text{in}} r)}{m_{eff}^{\text{in}} r} + \Phi_c$$

$$\Phi^{\text{out}}(r) \approx R(\Phi_c - \Phi_\infty)\exp(m_{eff}^{\text{out}} R) \times \frac{\exp(-m_{eff}^{\text{out}} r)}{r} + \Phi_\infty$$

Notice that due to the exponential factor, the r dependence in $\Phi^{\text{in}}(r)$ is quite suppressed well inside the body (where $\Phi^{\text{in}}(r) \approx \Phi_c$), and only within a *thin shell* near the surface of the body the field grows to match the exterior solution $\Phi^{\text{out}}(r)$.

Figure 1 depicts the two body problem considered below. In order to solve it, first we expand the most general solution for the two body problem in complete sets of solutions inside and outside the test body:

$$\Phi(r,\phi,\theta) = \begin{cases} \sum_{lm} C^{in}_{lm} i_l(\mu r) Y_{lm}(\theta,\phi) & r \leq R_2 \\ \sum_{lm} C^{out}_{lm} k_l(\tilde{\mu} r) Y_{lm}(\theta,\phi) & r > R_2 \end{cases} \tag{3}$$

where $i_l(\mu r)$ and $k_l(\tilde{\mu} r)$ are the spherical modified Bessel functions; $\tilde{\mu} = m^{out}_{eff}$ and $\mu = m^{\text{test body}}_{eff}$; and $R_2 = R_{\text{test body}}$.

Now we assume that the chameleon field outside the test body can be taken as the one body problem solution:

$$\Phi^{\text{out}}(\rho) = C\frac{\exp(-\tilde{\mu}\rho)}{\rho} + \Phi_\infty$$

where $\boldsymbol{\rho} = \boldsymbol{r} - D\hat{z}$.

We shall use the following identity to rewrite $\Phi^{\text{out}}(\rho)$ in terms of the coordinate system centered in the middle of the test body:

$$\frac{\exp(-\tilde{\mu}|\boldsymbol{r}_2 - \boldsymbol{r}_1|)}{4\pi|\boldsymbol{r}_2 - \boldsymbol{r}_1|} = \tilde{\mu}\sum_{l=0}^{\infty} i_l(\tilde{\mu}r_2)k_l(\tilde{\mu}r_1)\times \sum_{m=-l}^{l} Y_{lm}(\theta_2,\phi_2)Y^*_{lm}(\theta_1,\phi_1)$$

where $r_2 < r_1$. For the problem at hand $\boldsymbol{r}_1 = D\hat{z}$, $\theta_1 = 0$, $\boldsymbol{r}_2 = \boldsymbol{r}$.The problem we are considering is symmetric respect to rotations around the z axis, and therefore, only the term with $m = 0$ contributes to the solution. Now we match both solutions as follows:

$$\Phi_{\text{in}}(R_2) = \Phi_{\text{out}}(R_2)$$

and find:

$$C_{l0} = \left[(\Phi_\infty - \Phi^{2\text{min}}_{\text{in}})\delta_{l0} + C\tilde{\mu} i_l(\tilde{\mu}R_2)k_l(\tilde{\mu}D)\right]\times \frac{\sqrt{4\pi(2l+1)}}{i_l(\mu R_2)} \tag{4}$$

where $\Phi^{2\text{min}}_{\text{in}}$ is now the value of Φ that makes the effective potential of the two body problem reach its minimum inside the test body.

Table 1. Values of n (parameter of the chameleon potential) and β intervals (coupling of the chameleon to matter) such that the Sun and the Earth satisfy the thin shell condition ($M = 1000.[\text{cm}^{-1}]$).

n	β for the Sun	β for Earth
1	$(10^{-13}, 10^{-1})$	$(10^{-11}, 10^{-1})$
2	$(10^{-16}, 10^{-1})$	$(10^{-13}, 10^{-1})$
3	$(10^{-17}, 10^{-1})$	$(10^{-14}, 10^{-1})$
4	$(10^{-19}, 10^{-1})$	$(10^{-15}, 10^{-1})$
5	$(10^{-20}, 10^{-1})$	$(10^{-16}, 10^{-1})$

3. Force on a free falling body

In this section we calculate the force on a free falling test body using the approximate solution for the chameleon field we have found in section 2.

$$\begin{aligned} F_z &= -\int_V d^3x T\frac{\beta}{M_{pl}}e^{\frac{\beta}{M_{pl}}\Phi}\boldsymbol{\nabla}\Phi \\ &= -T\int_V d^3x \frac{\partial \exp[\frac{\beta\Phi}{M_{pl}}]}{\partial z} \\ &= -2\pi T\int_0^{R_2}\rho d\rho\left[e^{[\frac{\beta}{M_{pl}}\Phi(Y+\rho)]} - e^{[\frac{\beta}{M_{pl}}\Phi(Y-\rho)]}\right] \\ &\simeq -\frac{2\pi T\beta}{M_{Pl}}\int_0^{R_2}\rho d\rho[\Phi(Y+\rho) - \Phi(Y-\rho)] \end{aligned} \tag{5}$$

being $Y = D{+}R_2$. Now we evaluate the field using Eqs. (3) and (4) considering only the term with $l = 0$:

$$F_z \simeq \frac{4\pi T\beta}{M_{pl}}\left[\left(\frac{C\exp(-\tilde{\mu}D)}{\tilde{\mu}R_2 D} - \left(\Phi^{2\text{min}}_{\text{in}} - \Phi_\infty\right)\right)\times \frac{R_2\sinh(\mu(D+R_2))}{\mu}\right] \tag{6}$$

From the expression above, it follows that the force has an important dependence with the distance between the test body and the large body through the term $\sinh(\mu(D{+}R_2))$. Now we can compare the predictions of the chameleon model using Eq. (6) with experimental bounds on the differential acceleration of two bodies of different composition: $\eta = 2\frac{|a_1-a_2|}{|a_1+a_2|}$. For the case of the test body we must consider

Table 2. Results of the Eötvös experiments (for original references see Kraiselburd & Vucetich 2012). Columns 1 and 2 show the composition of the bodies that are free-falling, column 3 indicates whether the experiment measures the free fall to the Earth or Sun, column 4 shows the experimental bound on η.

Body 1	Body 2	source	$(\eta \pm \sigma_\eta) \times 10^{11}$
Al	Au	Sun	1.3 ± 1.5
Al	Pt	Sun	0.03 ± 0.04
Si/Al	Cu	Sun	0.51 ± 0.67
Moon-Like	Earth-Like	Sun	0.005 ± 0.089
Be	Ti	Sun	- 0.031 ± 0.045
Be	Al	Sun	0.0 ± 0.042
Be	Ti	Earth	0.003 ± 0.018
Be	Al	Earth	-0.015 ± 0.015
Be	Al	Earth	-0.02 ± 0.28
Be	Cu	Earth	-0.19 ± 0.25

$T = -\rho + 3P$ since in this case $\rho \simeq 10^{11}\mathrm{cm}^{-4}$ and $3P \simeq 10^{8}\mathrm{cm}^{-4}$. Table 1 shows the values of the coupling of the chameleon to matter β for each value of the parameter of the potential n such that the large body satisfies the thin shell condition. For the values shown in table 1 we fixed $M = 1000[\mathrm{cm}^{-1}]$ (the other free parameter of the chameleon potential), but we have also calculated β for lower values of $M = 1, 10, 100[\mathrm{cm}^{-1}]$; for these cases the minimum value for β to satisfy the thin shell condition are larger than the values shown in table 1 and thus we the corresponding values of η are larger than current experimental bounds. . Table 3 shows the value of η for pairs of test bodies with different composition free-falling in the gravitational field of the earth; the experimental bounds for the same test bodies are shown in Table 2. We have also calculated η for experiments studying free falling bodies towards the sun; in this case the difference between the accelerations of the test bodies is so large that $\frac{a_1}{|a_1+a_2|} \simeq 1$ and $\frac{a_2}{|a_1+a_2|} \simeq 0$ and therefore in all these situations we can use $\eta = 2$. We obtain similar results for the bodies falling through the earth (see table 3).These results suggest that the

Table 3. Predictions for the differential acceleration of free-falling bodies in the gravitational field of the earth within the approximation considered in this paper. Columns 1 and 2 show the composition of the bodies that are free falling, column 3 shows the value of n (free parameter of the chameleon potential), column 4 shows the value of β (coupling of the chameleon to matter) and column 5 shows the value of the differential acceleration η.

Body 1	Body 2	n	β	$\eta_{chameleon}$
Be	Ti	1	$(10^{-10}, 10^{-5})$	2.000
		1	10^{-11}	1.939
		2	$(10^{-12}, 10^{-6})$	2.000
		2	10^{-13}	1.860
		3	$(10^{-13}, 10^{-7})$	2.000
		3	10^{-14}	1.944
		4	$(10^{-14}, 10^{-8})$	2.000
		4	10^{-15}	1.850
		5	$(10^{-14}, 10^{-8})$	2.000
		5	10^{-15}	1.996
		5	10^{-16}	1.498
Be	Al	1	$(10^{-10}, 10^{-5})$	2.000
		1	10^{-11}	1.231
		2	$(10^{-11}, 10^{-6})$	2.000
		2	10^{-12}	1.982
		2	10^{-13}	1.059
		3	$(10^{-12}, 10^{-7})$	2.000
		3	10^{-13}	1.992
		3	10^{-14}	1.287
		4	$(10^{-13}, 10^{-8})$	2.000
		4	10^{-14}	1.959
		4	10^{-15}	1.060
		5	$(10^{-14}, 10^{-8})$	2.000
		5	10^{-15}	1.716
		5	10^{-16}	0.686
Be	Cu	1	$(10^{-11}, 10^{-5})$	2.000
		2	$(10^{-12}, 10^{-6})$	2.000
		2	10^{-13}	1.998
		3	$(10^{-14}, 10^{-7})$	2.000
		4	$(10^{-14}, 10^{-8})$	2.000
		4	10^{-15}	1.997
		5	$(10^{-15}, 10^{-8})$	2.000
		5	10^{-16}	1.994

chameleon model might be ruled out by Eötvös type experiments. Since we are working with an approximate solution of the chameleon field equation, we avoid reaching to stronger conclusions. Work in progress includes finding an exact solution of the two body problem to verify results shown in this paper.

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