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Disformal coupling, CMB spectral distortion and distance duality relation

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Abstract. Light scalar fields can naturally couple disformally to Standard Model fields without giving rise to the unacceptably large fifth forces usually associated with light scalars. We show that these scalar fields can be studied and constrained through their interaction with photons, and focus particularly on changes to the Cosmic Microwave Background spectral distortions and violations of the distance duality relation. We then specialise our constraints to scalars which could play the role of axionic quintessence. The work here presented was done in collaboration with P. Brax, C. Burrage and A. C. Davis.

Key words. varying alpha, varying speed of light, disformal coupling, axions

1. Introduction

New, light scalar degrees of freedom appear naturally in most attempts to explain the current acceleration of the expansion of the Universe, but their presence gives rise to fine tuning problems. Firstly, if a new light scalar field couples to matter fields it is expected that it will mediate a new, long range fifth force. In order to be in agreement with terrestrial and solar system measurements we either need a reason why such couplings between the scalar field and matter are forbidden, or we need to make the theory non-linear in such a way that it has a screening mechanism which dynamically suppresses the effects of the scalar force. Secondly, if these scalar fields are related to the mechanism driving the accelerating expansion of the Universe, we expect them to have Compton wavelengths corresponding to distance scales similar to the size of the observable Universe today.

One simple solution is provided by assuming that the scalar field only has disformal coupling to matter . These interactions were first discussed by Bekenstein (1993), who showed that the most general metric that can be constructed from $g_{\mu\nu}$ and a scalar field that respects causality and the weak equivalence principle is:

$$\tilde{g}_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi \ , \tag{1}$$

where the first term gives rise to 'conformal' couplings between the scalar field and matter, and the second term is the 'disformal' coupling. Here $X = (1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. As we will see the disformal interactions give rise to Lagrangian interaction terms of the form $\mathcal{L} \supset \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$, where $T^{\mu\nu}$ is the energy momentum tensor of matter fields. At the classical level, in static configurations, the coupling to the matter energy density vanishes annulling any fifth force which only exists in dynamical situations. Moreover, the interactions controlled by this coupling involve two copies of the scalar field, and at least two matter particles. Therefore the first scalar corrections to two particle scattering must be at the one-loop level and so this type of coupling for the scalar field does not give rise to any classical forces in vacuum.

We find that light rays in these models follow geodesics where the speed of light is not constant anymore. This implies that distance measurements are affected and more particularly the distance duality relation. When adding a direct coupling to electromagnetism and therefore a change in time of the fine structure constant, we find that the duality relation receives additional corrections associated to this. Also the amplitude of the CMB spectrum is modified by the same quantity as the distance duality relation. This can be constrained as it leads to a μ distorsion of the CMB spectrum which is precisely bounded since last scattering. We can therefore give new bounds on a combination of the variation of the speed of light and the fine structure constant from last scattering and from a redshift $z \sim 1$. When complementary bounds on the fine structure constant are available, this provides independent constraints on the variation of the speed of light, typically from $z \sim 1$ at the percent level.

2. Disformally coupled scalar fields - Gravity and matter sectors

We consider the coupling of a scalar field to matter governed by the action

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{2\kappa_4^2} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_m(\psi_i, \tilde{g}_{\mu\nu}) \,, \tag{2}$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{2}{M^4}\partial_\mu\phi\partial_\nu\phi$. The coupling scale M is constant and unknown and should be fixed by observations. The metric $\tilde{g}_{\mu\nu}$ is the Jordan frame metric with respect to which matter is conserved. On the other hand, the metric $g_{\mu\nu}$ defines the Einstein frame and in this frame energy-momentum is not conserved.

In what follows we will restrict ourselves to the leading order effects of the disformal coupling between the scalar field and matter. Therefore we calculate only to leading order in $1/M^4$, implying that the action can be expanded as

$$S = \int d^4x \sqrt{-g}\left(\frac{R}{2\kappa_4^2} - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}\right) + S_m(\psi_i, g_{\mu\nu}) \,. \tag{3}$$

We see than that the new coupling to matter involves two derivatives and can only be probed in the presence of matter when dynamical situations are considered.

3. Coupling with radiation sector

We generalize the situation slightly by introducing a field dependent coupling constant, controlled by a new unknown scale Λ, so that the kinetic term for photons contains

$$S_{\rm rad} \supset -\int d^4x \sqrt{-\tilde{g}}\frac{1}{4}\left(1 + \frac{4\phi}{\Lambda}\right)F^2 \,, \tag{4}$$

where contractions are made with the Jordan frame metric. This interaction between the scalar field and photons is similar to that of an axion. In particular, the fine structure constant becomes field dependent: $\alpha(\phi) = \frac{\alpha_\star}{1+\frac{4\phi}{\Lambda}}$, where $\alpha_\star$ is its value in the absence of coupling.

To leading order in $1/M^4$ the photon Lagrangian becomes

$$\mathcal{L} = \sqrt{-g}\left(-\frac{1}{4}F^2 - \frac{\phi}{\Lambda}F^2 + \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}_{(\gamma)}\right) \,, \tag{5}$$

where $T^{\mu\nu}_{(\gamma)} = F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{g^{\mu\nu}}{4}F^2$ is the Einstein frame energy-momentum tensor of the photon. In the following we show that making the photon coupling constant (and therefore the fine-structure constant) scalar field dependent means that the Jordan frame photon energy-momentum tensor is not conserved, while making the effective metric 'seen' by photons scalar field dependent modifies their geodesics. For details see Brax et al. (2013).

3.1. Maxwell's equation and photon (non)-conservation

The equation of motion resulting from the Lagrangian in Equation (5) gives the generalised form of Maxwell's equation

$$\partial_\alpha\left[\left(1+\frac{4\phi}{\Lambda}+\frac{1}{M^4}(\partial\phi)^2\right)F^{\alpha\beta}\right]-\frac{2}{M^4}\partial_\alpha\left[\partial^\mu\phi\left(\partial^\alpha\phi F^\beta_\mu-\partial^\beta\phi F^\alpha_\mu\right)\right]=0\ . \tag{6}$$

Considering only time variations of the scalar field and working in the conformal Lorentz gauge where $\partial_\alpha A^\alpha = 0$ and $A_0 = 0$ we find

$$-\partial_0[C^2(\phi,\phi')\partial_0 A^i]+D^2(\phi,\phi')\Delta A^i=0\ , \tag{7}$$

where $\Delta = \partial_i\partial_i$, the index i runs only over spatial directions, and

$$C^2(\phi,\phi')=1+\frac{4\phi}{\Lambda}+\frac{1}{M^4a^2}\phi'^2,\quad D^2(\phi,\phi')=1+\frac{4\phi}{\Lambda}-\frac{1}{M^4a^2}\phi'^2\ , \tag{8}$$

where $' = \partial_0$ is the derivative in conformal time η with $ds^2 = a^2(-d\eta^2 + dx^2)$. It is possible to show that light rays follow geodesics of $\tilde{g}_{\mu\nu}$ (see Brax et al. (2013)). Along these geodesics the speed of light varies, which is apparent in the phase of the solution to Maxwell's equation.

If C and D are close to one and vary over cosmological times, we find that in the sub-horizon limit the dispersion relation is

$$\omega^2=c_p^2(\eta)k^2,\qquad c_p^2=(D(\phi,\phi')/C(\phi,\phi'))^2=1-\frac{2}{M^4a^2}\phi'^2\ . \tag{9}$$

The energy momentum tensor in the Jordan frame gives the energy density $\tilde{\rho}_\gamma = -\tilde{T}^{(\gamma)0}_0 = \frac{A^2k^2}{2a^4}$, which satisfies the conservation equation $\dot{\tilde{\rho}}_{(\gamma)} + 4H\tilde{\rho}_{(\gamma)} = -\frac{4}{\Lambda}\dot{\phi}\tilde{\rho}_{(\gamma)}$, implying that

$$A^2=A_0^2e^{-\frac{4}{\Lambda}(\phi-\phi_0)}=A_0^2\frac{\alpha}{\alpha_0}\ , \tag{10}$$

to leading order in ϕ/Λ, where A_0, ϕ_0 and α_0 are constants of integration. Therefore the photon intensity varies along photon trajectories.

4. Distance Duality Relations

There are two types of distances that can be inferred from observations that are commonly used in cosmography. The angular diameter distance $d_A^2 = \frac{dS_{\rm emit}}{d\Omega_{\rm obs}}$ of an object is obtained by considering a bundle of geodesics converging at the observer under a solid angle $d\Omega_{\rm obs}$ and coming from a surface area $dS_{\rm emit}$. The luminosity distance is given in terms of the emitter luminosity $L_{\rm emit}$ and the radiation flux received by the observer $F_{\rm obs}$ by $d_L^2 = \frac{L_{\rm emit}}{4\pi F_{\rm obs}}$. The luminosity and angular distances are related in the standard cosmology by Etherington's theorem (Ellis 2009), also known as distance duality relation:

$$d_L(z)=(1+z)^2d_A(z)\ . \tag{11}$$

The validity of the distance duality relation requires that photons propagate along null geodesics and that the geodesic deviation equation holds. In addition, the number of photons must be conserved.

It turns out (Brax et al. 2013) that the duality relation is modified by a function τ

$$d_L = \tau(\eta_{\rm obs}, \eta_{\rm emit})(1+z)^2 d_A \ , \tag{12}$$

where

$$\tau(\eta_{\rm obs}, \eta_{\rm emit})^2 = \left(\frac{\alpha_{\rm emit}}{\alpha_{\rm obs}}\right)\left(\frac{c_{\rm obs}}{c_{\rm emit}}\right)^2 \ . \tag{13}$$

If both the luminosity distance and the angular diameter distance can be measured as a function of redshift, then a lack of violation of the duality relation can be used to constrain the interactions of the disformal scalar field with photons.

5. CMB μ distortion

Our understanding of the primordial Universe leads us to expect that the CMB will display an almost perfect black body spectrum:

$$I(k, \eta_i) = \frac{k^3}{e^{k/T_0} - 1} \ , \tag{14}$$

Assuming that the only distortions appear through the influence of the scalar field as the light propagates towards us from the time of last scattering, then the measured spectrum will be:

$$I_{\rm obs}(k, \eta) = \left(\frac{d_A}{r_{\rm emit}}\right)^2 G(k, \eta, \eta_i) I(k, \eta_i) \ . \tag{15}$$

The first factor is a geometrical factor depending on the way the reciprocity relation is modified by the variation of the speed of light (Ellis et al. 2013), the second factor G appears because of the attenuation of the amplitude due to the change of the fine structure constant and the exchange of energy with the scalar field. It is easily found computing the radiation intensity as:

$$I = \frac{1}{2}[(\partial_0 A^i)^2 + B_i B^i] = \frac{\omega(k,\eta_i)}{\omega(k,\eta)} \frac{C^2(\eta_i)}{C^2(\eta)} \frac{k^2 + \omega^2(k,\eta)}{k^2 + \omega^2(k,\eta_i)} I(k,\eta_i) \equiv G(k,\eta,\eta_i) I(k,\eta_i) \ , \tag{16}$$

The combined effect is therefore

$$I_{\rm obs}(k, \eta) = \tau^{-2}(\eta, \eta_i) I(k, \eta_i) \ , \tag{17}$$

where the function τ was defined in equation (13), a result which is valid on subhorizon scales and when the speed of light varies cosmologically. So we can rewrite the intensity as

$$I_{\rm obs}(k, \eta) = \frac{k^3}{e^{k/T_0 + \mu} - 1} \ , \tag{18}$$

where the adimensional chemical potential is given by

$$\mu = -2(e^{-k/T_0} - 1)\delta\tau \ , \tag{19}$$

where $\tau(\eta, \eta_i) = 1 + \delta\tau$. Hence we have found that the CMB spectrum and the duality relation are distorted due to the same τ function whose origin follows from both the disformal coupling and direct coupling of a scalar field to photons.

6. Observational constraints

6.1. μ distortion constraints

The present limits on the amount of μ distortion in the CMB spectrum come from COBE/FIRAS observations. At 95% c.l. they are $|\mu| < 3.3 \times 10^{-4}$ at wavelengths of cm and dm (Mather et al. 1994). The proposed experiment PIXIE (Kogut at al. 2011) will be sensitive to $\mu \sim 10^{-8}$.

Assuming that the constraining power of observations of the black body spectrum of the CMB comes from observations at frequencies corresponding to $T_0 \sim 2.7K$ (Kogut at al. 2011), we find

$$|\mu| < 3.3 \times 10^{-4} \Rightarrow |\delta\tau| < 2.6 \times 10^{-4} \ . \qquad (20)$$

Notice that μ is linearly dependent on $\delta\tau$. So the four-orders-of-magnitude improvement expected from PIXIE would translate into a constraint on $|\delta\tau|$ at the level of 10^{-8}.

6.2. Distance Duality constraints

We need distance duality relation constraints to be as independent as possible from the cosmological model, so that we can use them in our framework, without worrying about the effects of the scalar field on the evolution of the Universe. The best current constraint of this kind is provided by Holanda et al. (2012). They compare galaxy cluster mass fraction estimates obtained from X-ray measurements (which probe d_L/d_A) and observations of the Sunyaev-Zeldovic effect (which probe d_A). The clusters considered are all in the redshift range $z \in (0.1, 0.9)$.

The constraint found in Holanda et al. (2012) can be easily translated into a constraint on $\tau(z) \equiv \tau(\eta_{obs}, \eta_{emit})$, where we put $z_{obs} = 0$ and $z_{emit} = z$ (Brax et al. 2013):

$$\tau(z = 0.35) = 0.979 \pm 0.056$$

at 2σ, which can also be stated as $\delta\tau(z = 0.35) = -0.021 \pm 0.056$.

Since present constraints on $\frac{\delta\alpha}{\alpha}$ coming from Quasar observations are at least three order of magnitude stronger than these (Murphy et al. 2001), we can just assume that all the contribution to $\delta\tau$ comes from the speed of light variation, and so at 68% c.l. we set:

$$\left|\frac{\delta c_p}{c_p}\right| < 0.060 \ . \qquad (21)$$

7. Constraints on Axionic quintessence models

Disformal couplings will naturally arise in dark energy models which possess an axionic shift symmetry, such models have also been termed 'thawing quintessence' because of the dynamics of the scalar field. They are typically described by a scalar field with potential

$$V(\phi) = \frac{\Lambda_0^4}{2}\left(1 + \cos\frac{\phi}{f}\right) , \qquad (22)$$

where Λ_0 and f are constant model parameters. It is assumed that initially $\phi_i \ll f$ and that the field only starts rolling at late times in the history of the universe (Brax et al. 2013).

We derive constraints on the axionic quintessence model using the information on variation of α and variation of the speed of light as discussed in the previous Section. The constraint on $\delta c_p/c_p$ refers to variation between redshift $z_e \sim 0.35$ (average redshift of measured clusters) and now. The constraint on $\delta\alpha/\alpha$ refers to variation between redshift $z_e \sim 1$ and now. Slow rolling starts at redshift z_r such that $z_r < 1$. In the following we assume that it actually starts after $z = 0.35$. For a more general case see Brax et al. (2013).

The variations in fine structure constant and the speed of light as a function of the scalar field and cosmological parameters are, if slow-roll starts after radiation emission (Brax et al. 2013):

$$\frac{\delta\alpha}{\alpha} = 4\sqrt{6}\frac{\sqrt{1+w_\phi}}{\Lambda}\sqrt{\frac{\frac{3}{2}\Omega_{\Lambda 0}m_P^2}{(1+z_r)^3(1-\frac{3}{2}(1+w_\phi))}}, \qquad \frac{\delta c}{c} = \frac{4(1+w_\phi)H_0^2\Omega_{\Lambda 0}m_P^2}{M^4(1+z_r)^3(1-(1+w_\phi))} \tag{23}$$

Using the constraint on $\delta\alpha/\alpha$ from Quasars observations at redshift $z \sim 1$ we get a constraint on a combination of Λ, z_r and the equation of state parameter w_ϕ:

$$\frac{4\sqrt{6}}{\Lambda}\sqrt{1+w_\phi}\sqrt{\frac{\frac{3}{2}\Omega_{\Lambda 0}m_{\rm Pl}^2}{(1+z_r)^3[1-\frac{3}{2}(1+w_\phi)]}} < 0.81\cdot 10^{-5}\ . \tag{24}$$

Using the constraint (21) on $\delta c_p/c_p$ we can derive limits on a combination of M^4 and z_r

$$\left(\frac{M}{10^{-2}\ \mathrm{eV}}\right)^4 > 0.13\frac{(1+w_\phi)}{(1-\frac{3}{2}(1+w_\phi))}\frac{1}{(1+z_r)^3} \tag{25}$$

For $w_\phi = -0.96$ and $z_r \to 0$ the constraints for the scalar field coupling constants are:

$$\Lambda > 6.6\times 10^{32}\mathrm{eV}, \qquad M^4 > 5.6\times 10^{-11}\mathrm{eV}^4. \tag{26}$$

8. Conclusion

Disformal couplings to matter naturally arise for scalar fields with shift symmetries, such as those suggested to explain the late time acceleration of the expansion of the Universe. We have shown that such interactions can be constrained with cosmological observations, in particular with observations of the spectral distortions of the CMB and from tests of the distance duality relation. We have shown that both are affected by a disformal and a direct coupling to photons. The former leads to a variation of the speed of light and the latter a variation of the fine structure constant. We discussed constraints on axionic quintessence models.

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