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# Cumulative propagation of physical uncertainties in stellar models

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**Abstract.** We quantify the effects of the uncertainties in the main physical inputs on the evolution of low mass stars. We calculated several thousands of stellar tracks by simultaneously changing the main physical inputs within their current range of uncertainty. The analysis was conducted performing a systematic variation on a fixed grid, in a way to obtain a full crossing of the perturbed input values. We find that, for a 0.9 $M_{\odot}$ model, the cumulative uncertainty on the turn-off, the red-giant branch tip, and the ZAHB luminosities accounts for ±0.021 dex, ±0.03 dex, and ±0.045 dex respectively, while the central hydrogen exhaustion time varies of about ±0.72 Gyr.

**Key words.** stars: evolution – stars: horizontal-branch – stars: interiors – stars: low-mass – stars: Hertzsprung-Russell and C-M diagrams

## 1. Introduction

In order to understand the actual significance of the application of stellar models when deriving quantitatively fundamental stellar, and even cosmological, parameters, it is of primary importance to estimate the global uncertainty affecting these models caused by the still uncertain micro-physical inputs.

In the present paper we focus on stellar models of low-mass stars, from the main sequence (MS) to the zero age horizontal branch (ZAHB). Relying on the computation of several thousands of stellar models, we quantified the effects of the variations of the main physical inputs on: the turn-off luminosity, the central hydrogen exhaustion time, the luminosity and helium-core mass at the Red Giant Branch (RGB) tip, and the ZAHB luminosity in the RR Lyrae region at $\log T_{\rm eff} = 3.83$.

## 2. Description of the method

In this section we briefly describe the method, for a much more detailed and exhaustive treatment we refer to Valle et al. (2013). We focus on the evolution of a star with mass $M = 0.90$ $M_{\odot}$, initial metallicity $Z = 0.006$ (with heavy elements solar mixture by Asplund et al. 2009) and helium abundance, $Y = 0.26$.

The adopted stellar evolutionary code, FRANEC, has been extensively described in previous papers (Cariulo et al. 2004; Degl'Innocenti et al. 2008, and references therein). A detailed discussion of the recent updates of the physical inputs can be found in Valle et al. (2009) and Tognelli et al. (2011).

The code adopted here is the same used for the construction of the Pisa Stellar Evolution Data Base[1] for low-mass stars, as illustrated in Dell'Omodarme et al. (2012), where a detailed description of the inputs of the stellar evolutionary code and of the ZAHB construction technique can be found.

The selected physical inputs allowed to vary and their assumed uncertainty are listed in Table 1. We refer to Valle et al. (2013) for a detailed discussion on their uncertainty estimate.

We performed a technique that allows the exploration of the edge of the variability region, and that is robust in presence of interaction among the inputs of the calculations, since it does not assume physical independence of the individual processes. In fact, we performed a systematic variation of the inputs on a fixed grid. For each physical input, we introduced a three-values multiplier $p_i$ with value 1.00 for the reference case and values $1.00 \pm \Delta p_i$ for perturbed cases ($\Delta p_i$ is the uncertainty listed in Table 1), which defines the range of variation. For each stellar track calculation, a set of multiplier values (i.e. $p_1, \ldots, p_7$ for the seven input physics allowed to vary) is chosen and kept constant during the evolution of the stellar structure. In order to cover the whole parameters space, calculations of stellar tracks were performed for a full crossing, i.e. each parameter value $p_i$ was crossed with all the values of the other parameters $p_j$, with $j \neq i$.

In this way, we computed stellar models for all the possible sets of multiplier values. A total of $3^7 = 2187$ tracks were then computed, with same mass, chemical composition and mixing-length parameter $\alpha_{\rm ml}$ (i.e. 1.90).

## 3. Cumulative physical uncertainty in stellar models

Taking advantage of the very large set of stellar models covering all the possible combinations of simultaneously perturbed input physics, we quantified the cumulative physical uncertainty affecting low mass stellar models.

[1] `http://astro.df.unipi.it/stellar-models/`

**Table 1.** Physical inputs perturbed in the calculations and their assumed uncertainty.

| | | |
|---|---|---|
| $^1$H(p,$\nu e^+$)$^2$H reaction rate | $p_1$ | 3% |
| $^{14}$N(p,$\gamma$)$^{15}$O reaction rate | $p_2$ | 10% |
| radiative opacity | $p_3$ | 5% |
| microscopic diffusion velocities | $p_4$ | 15% |
| triple-$\alpha$ reaction rate | $p_5$ | 20% |
| neutrino emission rate | $p_6$ | 4% |
| conductive opacity | $p_7$ | 5% |

Fig. 1 shows the combined effect in the $\log L/L_\odot$ vs. $\log T_{\rm eff}$ plane of the variation of all the seven physical inputs (i.e. $p_1, \ldots, p_7$) listed in Table 1. This figure shows, for the first time, the error stripe associated to a stellar track, i.e. the region of the plane spanned by the perturbed stellar models. A detailed description of the technique employed for the construction of the error stripe is given in Valle et al. (2013).

We quantified also the global physical uncertainty affecting the current theoretical predictions of: the turn-off luminosity $L_{\rm BTO}$, the central hydrogen exhaustion time $t_{\rm H}$, the luminosity $L_{\rm tip}$ and the helium core mass $M_{\rm c}^{\rm He}$ at the RGB tip, and the ZAHB luminosity in the RR Lyrae region $L_{\rm HB}$.

**Table 2.** Total range of variation and range half-width of the theoretical predictions for the selected quantities for our reference case, i.e. $M = 0.90\ M_\odot$ with $Z = 0.006$ and $Y = 0.26$, due to input physics uncertainties.

| quantity | variation range | half-width |
|---|---|---|
| $\log L_{\rm BTO}$ | [0.334 - 0.376] dex | 0.021 dex |
| $t_{\rm H}$ | [9.83 - 11.26] Gyr | 0.72 Gyr |
| $\log L_{\rm tip}$ | [3.38 - 3.44] dex | 0.03 dex |
| $M_{\rm c}^{\rm He}$ | [0.4796 - 0.4879] $M_\odot$ | 0.0042 $M_\odot$ |
| $\log L_{\rm HB}$ | [1.52 - 1.61] dex | 0.045 dex |

Table 2 shows the total range of variation in predictions of the selected quantities for our reference stellar track of $M = 0.9\ M_\odot$ due to current input physics uncertainties. The turn-off log luminosity $\log L_{\rm BTO}$ varies in the range [0.334 - 0.376] dex (range half-width 0.021

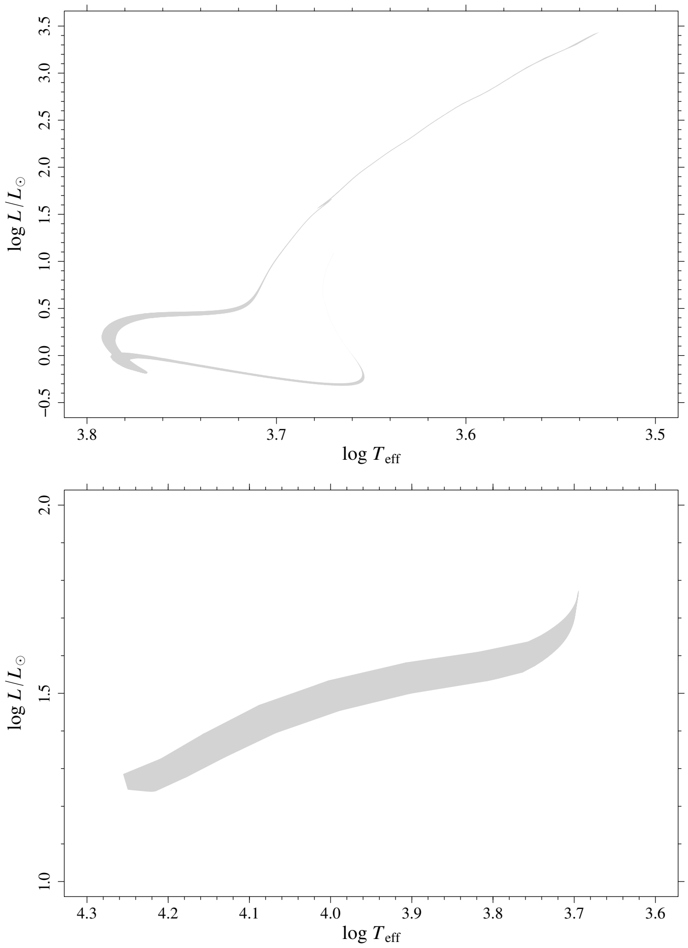


**Fig. 1.** Top panel: HR diagram showing the error stripe due to the variation of all the seven analyzed physical inputs (i.e. $p_1, \ldots, p_7$ in Table 1) on the stellar track with $M = 0.9\ M_{\odot}$, $Z = 0.006$, $Y = 0.26$ from pre-main sequence to helium flash. The narrowing of the error stripe in the RGB is due to the fact that the perturbed stellar models are disposed along the tracks itself. Bottom panel: as in the left panel, but for the ZAHB.

dex, $\approx$ 6% of the value obtained with unperturbed physical inputs). The total range of variation of the predicted central hydrogen exhaustion time $t_{\rm H}$ is [9.83 - 11.26] Gyr (0.72 Gyr, $\approx$ 6.5%). The RGB tip $\log L_{\rm tip}$ and ZAHB $\log L_{\rm HB}$ log luminosities vary, respectively, in the ranges [3.38 - 3.44] dex (0.03 dex, $\approx$ 1%) and [1.52 - 1.61] dex (0.045 dex, $\approx$ 3%). Finally, the helium core mass at the RGB tip $M_{\rm c}^{\rm He}$ varies in the range [0.4796 - 0.4879] $M_{\odot}$ (0.0042 $M_{\odot}$, $\approx$ 0.85%).

## 4. Conclusions

In this paper we tried to quantify the cumulative propagation of physical uncertainties in current generation of stellar models of low mass stars from the main sequence to the zero age horizontal branch. In the last two decades, several works studied the impact of uncertainties in physical inputs on stellar models by varying a single input at a time (see e.g. Chaboyer et al. 1995; Cassisi et al. 1998;

Castellani & Degl'Innocenti 1999; Castellani et al. 2000; Imbriani et al. 2001; Prada Moroni & Straniero 2002; Salaris et al. 2002; Imbriani et al. 2004; Weiss et al. 2005; Prada Moroni & Straniero 2007; Valle et al. 2009; Tognelli et al. 2011). As described in detail in Valle et al. (2013), we decided to follow a different approach, i.e. a systematic and simultaneous variation on a fixed grid within their current range of uncertainty of the main physical inputs adopted in stellar codes. Clearly such an approach is very expensive from the computational point of view but has the advantage with respect to the previous one to be more robust against possible interactions among the varied input physics, as any a priori independence among them is assumed.

Relying on a set of stellar models fully covering all the possible combinations of simultaneously perturbed input physics, we were able to compute the error stripe associated to a stellar track of $M = 0.90\ M_{\odot}$ with initial metallicity $Z = 0.006$ and helium abundance $Y = 0.26$, from the pre-main sequence up the ZAHB. As far as we know, this is the first time that an error stripe is computed and plotted for stellar tracks.

We quantified also the extension of the global variability regions for the turn-off luminosity $L_{\rm BTO}$, the central hydrogen exhaustion time $t_{\rm H}$, the luminosity $L_{\rm tip}$ and the helium core mass $M_{\rm c}^{\rm He}$ at the RGB tip, and the ZAHB luminosity in the RR Lyrae region $L_{\rm HB}$ (see e.g. Table 2). The turn-off log luminosity $\log L_{\rm BTO}$ varies of ±0.021 dex, while the RGB tip $\log L_{\rm tip}$ and ZAHB $\log L_{\rm HB}$ ones of ±0.03 dex and ±0.045 dex, respectively. The predicted central hydrogen exhaustion time $t_{\rm H}$ varies of ±0.72 Gyr, whereas the helium core mass at the RGB tip $M_{\rm c}^{\rm He}$ of $\pm 0.0042\ M_{\odot}$.

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