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A new point of view in the analysis of equilibrium and dynamical evolution of globular clusters

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Abstract. We develop models of globular clusters (GCs) with a different approach by applying thermodynamic principles to a Boltzmann distribution function, with an Hamiltonian function which contains an effective potential depending on the kinetic energy of the stars, due to the effect of tidal interactions induced by the hosting galaxy. The Hamiltonian function is solution of the Fokker-Planck equation solved in a different way with respect to the King approach. Interesting results implying a different caloric curve for the analysis of the evolution of GCs are presented.

Key words. Galaxy: globular clusters; evolution

1. Introduction

It is well known that dynamical evolution of GCs is strongly affected by collisions among the stars, making favourable a statistical mechanics approach in order to study this important phenomenon. GCs are stellar systems with masses within the interval $10^4 - 10^6\ M_\odot$, containing a number of stars of the order of 10^5. For their spherical symmetry, there is the possibility to test the evolution of a GC by studying a classical single mass King model (King 1966) in relation to thermodynamical instability phenomena.

In the evolution of GCs, stellar encounters strongly contribute in phase space mixing of stellar orbits and thermodynamics plays a centrale role in the gravitational equilibrium and stability of these clusters, being the average binary relaxation time shorter than their old absolute age which ranges between 10 to 13 Gyr.

On the other hand, the observations of the luminosity profiles of different GCs (King 1962) show similar curves depending only on different values of the star concentration, giving the possibility to fit them by an empirical law and suggesting a unique distribution function for the whole sample of clusters (King 1966).

Thus, the evolution of GCs can be simulated by a sequence of thermodynamic equilibrium configurations with different equilibrium parameters (Horwitz & Katz 1977), as small thermodynamic transformations which keep constant the functional form of the velocity distribution of stars like in the framework of Boltzmann statistical mechanics, with the relevant difference that the nature of collisions is described in the Fokker-Planck approximation.

The gravitational equilibrium models are completely equivalent to ones obtained by King in 1966, but in the analysis of the thermodynamical instabilities we shall find important innovations, relevant in the evolution of GCs.

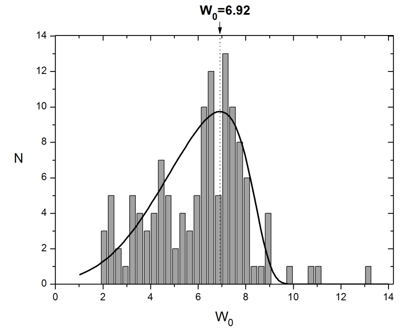

Fig. 1. MW GCs distribution at different values of W_0 based on the latest edition (2010) of the Harris Catalogue (Harris 1996).

2. The observed GC population and the new Fokker-Planck solution

The problem arises from the open question, not yet solved in the last forty years, on the distribution of Milky Way (MW) globular clusters (GC) with respect to critical point related to the onset of gravothermal catastrophe. This question was firstly outlined by Katz (1980) with the comparison of the value of the central gravitational potential (W_0=6.9) corresponding to the maximum of the GC distribution, with the value (W_0=7.4) corresponding to the onset of gravothermal catastrophe. These values are expected to be coincident because GCs had enough time to undergo the gravothermal catastrophe and, therefore, the distribution of GCs in terms of W_0 should peak exactly in correspondence to the critical value. In fact, the primeval Gaussian distribution, approaching the critical value during the evolution, deforms in a non-symmetric Gaussian curve due to the effect of gravothermal catastrophe which progressively subtracts the collapsed GCs with values of W_0 larger than the critical value. For these reasons, the resulting distribution must present a maximum which corresponds to the onset of gravothermal instability. Nevertheless the values are significantly different and also the new data from Harris Catalogue on MW GCs (157 objects) do not contribute to the solution of the problem (see Fig.1).

In order to understand if the onset of gravothermal catastrophe is correctly performed, we reconsider the solution of the Fokker-Planck equation, starting from the assumption that the distribution in energy must have a Boltzmann form to correctly define the entropy as the logarithm of the statistical weight of macroscopic states. For solving the equation in terms of Hamiltonian of the single star, we consider the Fokker-Planck equation in the form introduced by Spitzer & Harm (1958)

$$\frac{d}{dx}\left[G(x)\left(\frac{dg(x)}{dx}+2xg(x)\right)\right]+\lambda x^2 g(x)=0\,, \quad (1)$$

where

$$G(x)=\frac{4}{\sqrt{\pi}x}\int_0^x e^{-y^2}y^2 dy\,, \quad (2)$$

while $x = v/\sqrt{2}\sigma$ and $\sigma = \sqrt{k\theta/m}$ are the dimensionless velocity and the Boltzmann velocity dispersion, respectively. The parameter λ is the fractional loss rate of the stellar evaporation suffered by the cluster (King 1965).

We search a solution of the form $g(x) = Ae^{-H(x)/k\theta}$, where $H(x) = H_0(x) + H_1(x)$ is the Hamiltonian of the single star and $H_1(x) = 0$ when $\lambda = 0$. So that, being $\lambda \ll 1$, we can write

$$\exp\left[-H_1(\lambda,x)/k\theta\right] \simeq 1-\frac{\lambda}{k\theta}\frac{\partial H_1}{\partial\lambda}\bigg|_{\lambda=0}\,. \quad (3)$$

Inserting $g(x)$ in Eq.(1), equating the terms with the same power of λ and neglecting the higher order terms to λ we obtain

$$\frac{\partial H_1}{\partial\lambda}\bigg|_{\lambda=0}=\frac{\sqrt{\pi}}{8}k\theta\left(e^{x^2}-1\right)\,, \quad (4)$$

and, finally, we get the Hamiltonian form as

$$H(x)=k\theta\left\{x^2-\ln\left[1-\frac{\sqrt{\pi}}{8}\lambda\left(e^{x^2}-1\right)\right]\right\}\,. \quad (5)$$

3. Discussion

With the resolution of the Fokker-Planck equation we obtained in the Hamiltonian of the distribution an additional term, named "effective

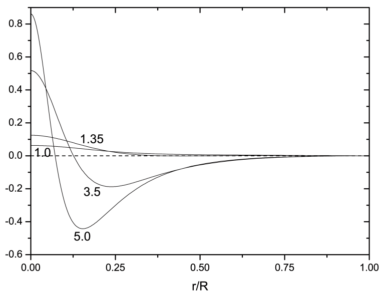

Fig. 2. Behaviour of the specific heat in function of the radial coordinate for different value of W_0.

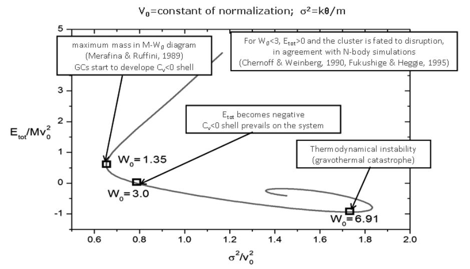

Fig. 3. Caloric curve taking into account the contribution of the effective potential.

potential", which is the effect of the equilibrium established by two competitive processes: one due to collisions among stars with an exchanging of energy which tends to modify the distribution by driving it to the Boltzmann one with the formation of a tail at large energy; one, in opposition, due to evaporation of stars as result of the presence of tidal forces induced by the hosting galaxy that removes continuously the stars and prevents the formation of the tail, maintaining a cutoff energy and a limited phase space. These two competitive effects keep unchanged the form of the energy distribution function during the dynamical evolution of the GCs, namely that we can consider as a "thermodynamical equilibrium" from the astrophysical point of view, with the collisions ruled by the Fokker-Planck approximation. The presence of the effective potential do not affect the internal dynamics of stars and sum of kinetic and gravitational energy remains constant in the motion between two consecutive collisions, as well the effective potential. The result is that the Hamiltonian (total energy) remains a constant of motion and the dynamical problem of the single mass orbit keep unchanged. Nevertheless, the presence of the effective potential has important consequences from the thermodynamical point of view resulting in the formation of regions with negative and positive specific heat which guarantee the evolution of the system towards the gravothermal instability (Merafina & Vitantoni 2014). These regions are shown in Fig.2.

Moreover, the additional (positive) contribution of the effective potential on the total energy $E_{tot} = E_{kin} + E_{gr} + E_{eff}$, considering also the virial condition $2E_{kin} + E_{gr} = 0$, implies that $E_{tot} = -E_{kin} + E_{eff}$. This term affects the caloric curve and change the critical point of the onset of gravothermal instability to a value which corresponds to the maximum of the GC distribution (from W_0=7.4 to W_0=6.9), as initially requested. The forty-year open question appears to be definitely solved with the presence of effective potential.

Another important result derived by this treatment is the threshold at $W_0 = 3$ corresponding to the zero total energy. Clusters with lesser values of W_0 are fated to disruption, because the tidal forces are prevalent to the gravitational ones. This result can explain observative data and N-body simulations going to the same conclusions (see e.g. Chernoff & Weinberg 1990; Fukushige & Heggie 1995).

All these results are summarized in Fig.3.

4. Effective potential and N-body simulation

Waiting for observative data on transverse velocities of stars in GCs, enabling to test the model and the presence of effective potential in real systems, we performed N-body simulations for two model with $N = 32768$ and $N = 262144$ stars with mass of $1M_{\odot}$. The core radius is $r_c = 10\,\mathrm{pc}$ and the value of the central gravitational potential is $W_0 = 5$ for both models, which differ only for the nuimber of stars. In order to extract informations valid for

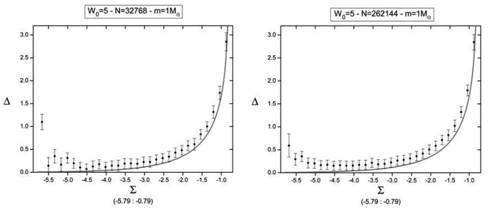

Fig. 4. Behaviour of the effective potential for two different N-body models.

the entire cluster we considered, for both models, the total energy of each star and calculated the total number of stars of equal sum of gravitational and kinetic energy, comparing the obtained distribution with the Boltzmann form with the Hamiltonian H above carried out. The effective potential can be expressed in dimensionless form as

$$\Delta = -\ln\left(1 - e^{\Sigma - W_\infty}\right) , \tag{6}$$

where W_∞ is the dimensionless form of the gravitational potential at the surface of the cluster depending only by W_0, while Σ is the dimensionless sum of the gravitational and kinetic energy of the single star. Thus, stars with the same value of Σ have the same value of the total energy expressend by the Hamiltonian H. Thes means that using the number of stars at fixed Σ allows to obtain the value of H. Therefore the effective potential is well obtained by subtracting kinetic and gravitational energy to the total energy calculated. The results are summarized in Fig.4.

We have a very good accordance with the predicted function. The small disagreement is due to the transition from continuous (theory) to discrete (N-body simulation) system, depending on the choice of the radial-energy mesh for individuating the sample of stars with same energy. This disagreement becomes more relevant at low energies, especially at the center of the cluster, for lack of stars at zero kinetic energy.

5. Conclusions

The additional contribution of the effective potential enables us to construct selfconsistent models admitting regions with positive and negative heat capacity which can exchange energy and produce gravothermal instability, without the necessity to assume an external bath as in the Lynden-Bell & Wood (1968) model.

We obtain a new critical value for the onset of gravothermal instability. This value coincides with one related to the peak of the GCs distribution, removing the unexpected difference outlined by Katz. This is an observational evidence of the presence of the effective potential, confirmed by the analysis of data of more than 150 GCs of the Harris Catalogue. The possibility to obtain also positive values of the total energy of the clusters, due to presence of the effective potential, gives the opportunity to explain the conditions of disruption of the clusters at very small values of W_0 where the tidal forces prevail on the gravitational ones, and are in complete accordance with the observations and N-body simulations.

We developed N-body simulations in order to test the presence of the effective potential. The presence is confirmed and also the behaviour is in very good accordance with the predicted one. The recent possibility of measuring transverse velocities of the stars in GCs opens important perspectives in order to test the presence of the effective potential also in real systems for supporting the validity of the model.

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