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Models of self-gravitating systems with short-range cutoff on the interactions

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Abstract. A class of spherically symmetric mean-field models of self-gravitating systems is defined by introducing a short-distance regularization of the gravitational interactions as an effective description of the small-scale physics regardless of its precise physical origin, yet retaining the lowered-Maxwellian ansatz on the distribution function as in the classic King model. We show that with these models it is possible to precisely recover all the solutions of the standard King model as well as to consistently describe highly concentrated density profiles. Preliminary results indicate that the latter are in good agreement with observed density profiles of some globular clusters deviating from the King model prediction in the inner regions. We argue that this approach may be a first step towards a unified theoretical description of a wide class of self-gravitating systems.

Key words. stars: kinematics and dynamics – methods: analytical – methods: numerical – globular clusters: general

1. Introduction

Many stellar systems ranging from star clusters to elliptical galaxies can be modeled, at least as a first approximation, as purely self-gravitating systems (SGSs). Spherically symmetric mean-field models as the King (1966) model are the simplest ones able to reasonably describe the observed density profile of a class of SGSs, i.e., non-collapsed globular clusters (GCs). However, these models are unable to account for higher central concentrations like those exhibited by post-core-collapse clusters or for the density profile of most elliptical galaxies. Moreover, high-resolution data made available in the last decade revealed high central concentrations also in some GCs previously considered as well described by King models: roughly 50% of the sample considered by Noyola & Gebhardt (2006) is not consistent with the flat core predicted by King models. In the following we shall consider King-like models where the gravitational interactions have been regularized at small distance such as to avoid the singularity of the potential. We shall demonstrate that the regularization produces, in addition to standard King flat-core profiles, also other classes of more centrally concentrated profiles. The latter profiles can be regarded as other "thermodynamic phases", although the system is out of thermal equilibrium.

Regularization can be achieved in many ways (e.g. softening the potential or including a hard-core repulsion); all involve a new length scale $a \ll R$, where R is the size of the system, such that the gravitational potential is modified for distances of order a. We shall refer to a as the cutoff length. Some kind of regularization is physically motivated by the fact that at very small length scales compared to R interactions, other than the gravitational ones, exist; but is also needed to make mean-field descriptions self-consistent (Nelson & Tremaine 1999) or to make the equations of motion numerically tractable in computer simulations. Hence, the regularization is typically considered as a (sometimes unavoidable) nuisance. Our point of view is different: we consider the insertion of a new length scale $a \ll R$ in the model as an effective way to take into account the small-scale physics that is lost when a mean-field model is built (star-star correlations, formation of binaries, and so on), regardless of its precise physical origin. Rather than being a quantity to be kept as small as possible so that it makes the smallest possible contribution to the physics, the small-scale cutoff a is a free parameter of the model.

2. Models with short-range cutoff

A first investigation of a class of King-like models with short-range cutoff was performed by Casetti & Nardini (2012). The gravitational interactions were regularized via a Plummer-like softening, i.e., the $1/r$ potential energy between two stars of mass m at distance r was replaced by

$$V_a(r) = -\frac{Gm^2}{\sqrt{r^2 + a^2}}, \qquad (1)$$

and then the lowered-Maxwellian form (King 1966) of the mass distribution function $f(r, v)$ was used. The potential (1) is not a Green function of the Laplace operator, so that there is no longer a Poisson equation and one has to solve a system of self-consistent integral equations to compute the mean-field potential $\varphi(r)$, from which the $f(r, v)$ itself, the density profile $\varrho(r) = \int d\mathbf{v}\, f(r, v)$ and all the collective physical quantities can be obtained. At a given value of a, the solutions are parametrized by the value W_0 of the dimensionless central potential, as in the King model. A convenient way to describe the collective physical behaviour is to plot the caloric curve, i.e., the global temperature as a function of the total energy. Although the system is not in thermal equilibrium one can still define a global temperature T in terms of the mean-field kinetic energy K (we set the Boltzmann constant to unity)

$$\frac{3T}{2} = K = \frac{1}{2}\int d\mathbf{r}\, d\mathbf{v}\, v^2 f(r, v) \qquad (2)$$

and a mean-field potential energy

$$U = \frac{1}{2}\int d\mathbf{r}\, \varrho(r)\, \varphi(r) \qquad (3)$$

so that the total energy is $E = K + U = 3T/2 + U$. Measuring distances in units of the tidal radius r_t and masses in units of the total mass M of the system one gets an energy scale GM^2/r_t so that one can define a dimensionless energy $\varepsilon = Er_t/(GM^2)$, a dimensionless temperature $\vartheta = Tr_t/(GM^2)$, and a dimensionless cutoff length $\alpha = a/r_t$. For the King model, i.e., when $\alpha = 0$, the caloric curve lies on a straight line with slope $-2/3$ as a consequence of the virial theorem. However, the King caloric curve has a limited support: in terms of the dimensionless variables ε and ϑ, one has $-2.15 \le \varepsilon \le -0.60$ and $0.40 \le \vartheta \le 1.42$. On the contrary, when $\alpha \neq 0$, the caloric curve extends down to the absolute minimum of the energy, given by $\varepsilon_{min} = -1/(2\alpha)$. In Fig. 1 the caloric curve obtained with $\alpha = 3.15 \times 10^{-3}$ is shown. The figure clearly shows that one of the consequences of the cutoff is to make states with much lower energy than in the case of the King model accessible. This notwithstanding, the high-energy part of the caloric curve is remarkably close to the King one, as shown in Fig. 2. Already with $\alpha \approx 3 \times 10^{-3}$ also the density profiles of the high-energy region are nearly indistinguishable from those obtained with the King model.

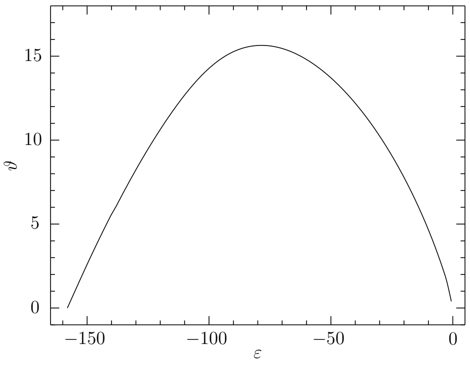

Fig. 1. Caloric curve with a cutoff $\alpha = 3.15 \times 10^{-3}$.

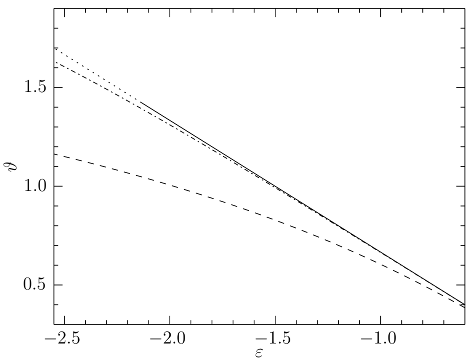

Fig. 2. High-energy region of the caloric curve with $\alpha = 3.15\times10^{-2}$ (dashed curve), $\alpha = 3.15\times10^{-3}$ (dot-dashed curve), $\alpha = 0$ (solid curve). The dotted line is the virial law.

2.1. Small cutoff and phase transition

Although the cutoff length α does not have a precise physical meaning in terms of a specific "microscopic" process, being only an effective description of the small-scale physics, one may safely argue that the value of α should be bounded between the size of a star and the average distance between stars. For a typical GC these bounds roughly translate into $10^{-9} \lesssim \alpha \lesssim 5 \times 10^{-2}$. Casetti & Nardini (2012) only explored the range $2 \times 10^{-3} \lesssim \alpha \lesssim 5 \times 10^{-2}$, that is, rather large values of α. Unfortunately, the numerical procedure employed by Casetti & Nardini (2012) encounters serious convergence problems at smaller α's. In order to overcome these difficulties, Lenzini & Casetti (2016) considered a different regularization of the potential, that is

$$\tilde{V}_a(r) = -\frac{Gm^2}{r}\left[1 - \exp\left(-\frac{r}{a}\right)\right] . \qquad (4)$$

At variance with the potential (1), the regularized potential (4) is the Green function of a differential operator (Alastuey 2013), so that the mean-field potential obeys a generalized fourth-order Poisson equation, of the form

$$\left(-a^2\triangle^2 + \triangle\right)\varphi(r) = 4\pi G\varrho\left[\varphi(r)\right] , \qquad (5)$$

where $\triangle = \nabla^2$ is the Laplace operator. Equation (5) can be solved with standard numerical methods and it is possible to reach values of α as small as 10^{-8} (see Lenzini & Casetti 2016 for details). Moreover, it is apparent that Eq. (5) becomes the Poisson equation obeyed by the King potential when $a \to 0$. For the values of α explored by Casetti & Nardini (2012), Lenzini & Casetti (2016) obtain the same results. For $\alpha \lesssim 1.8 \times 10^{-3}$ the caloric curve becomes singular: at an energy slightly smaller than the minimum King energy there is a jump in temperature (analogous to a first-order microcanonical phase transition) and, at an even smaller energy, a gap in energy opens up where no solutions are found (a "zero-order" phase transition). Hence there are three "thermodynamic phases" in the system: a high-energy phase corresponding to the King energies, an intermediate phase, and a low-energy phase separated from the other two phases by the energy gap. As α gets smaller, the energy gap becomes larger and the intermediate phase collapses onto the King one. Examples of density profiles associated to the three "phases" are plotted in Fig. 3 for $\alpha = 10^{-4}$. It is apparent that the short-range cutoff allows to obtain very different density profiles, ranging from flat-core King ones to highly collapsed ones, within the same model. Moreover, the flat-core King profiles are the same regardless of the value of α, provided $\alpha \lesssim 10^{-3}$.

3. Test with observed GC profiles

Since the King profiles are precisely recovered for any sufficiently small α, the model

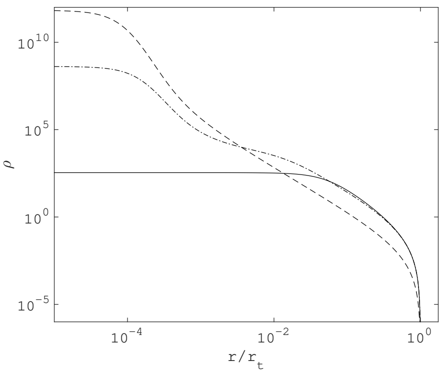

Fig. 3. Density profiles with $\alpha = 10^{-4}$: high-energy (King) phase (solid line), intermediate phase (dot-dashed curve), and low-energy collapsed phase (dashed curve).

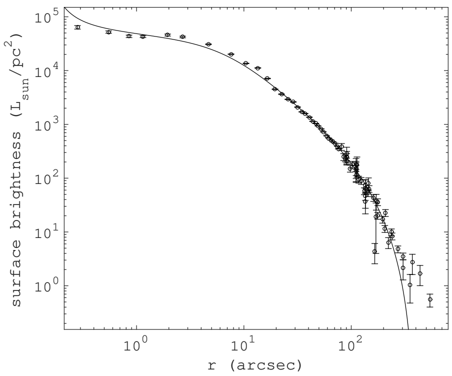

Fig. 4. Surface brightness profile of M 80 (points with errorbars, from Noyola & Gebhardt 2006 and Trager et al. 1995) compared to a theoretical profile with $\alpha = 10^{-4}$ and $W_0 = 19.5$ (solid line).

with short-range cutoff describes the density profiles of flat-core GCs equally well as the original King model without cutoff. We then made some preliminary tests of the predictions of the model against observed density profiles of GCs that exhibit significant deviations from a flat core in the inner regions. Datasets were constructed by merging high-resolution data obtained from HST observations by Noyola & Gebhardt (2006) at small radii ($r \lesssim 10^2$ arcsec) with ground-based data by Trager et al. (1995) at lager radii. As an example, the observed surface brightness profile of NGC 6093 (M 80) is plotted in Fig. 4, together with a (projected) theoretical profile belonging to the intermediate phase. The agreement is good. Similar results can be obtained for other GCs, for instance NGC 6624 and NGC 6293. Profiles belonging to the low-energy, collapsed phase reasonably agree with observations of collapsed GCs like NGC 7099 (M 15), although the inner brightness is systematically overestimated. Detailed tests will be presented elsewhere (Lenzini & Casetti 2016).

4. Conclusions

The models presented here do not aim at a detailed description of the observed properties of SGSs. Rather, they aim at showing that an effective description of the small-scale physics may be sufficient to predict very different density profiles within a single model. This may be a first step towards a unified theoretical description of a large class of SGSs in terms of simple models. The approach presented here can be in principle applied also to more refined distribution functions, as those used by Zocchi et al. (2012) or by Gieles & Zocchi (2015).

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