Mem. S.A.It. Vol. 89, 425
© SAIt 2018

Memorie della

The spatiotemporal aspects of SETI

A. Balbi

Dipartimento di Fisica, Università degli Studi di Roma “Tor Vergata”, Via della Ricerca Scientifica, 00133 Roma, Italy, e-mail: `balbi@roma2.infn.it`

Abstract. It is plausible that the distribution of (hypothetical) civilizations capable of interstellar communication, in our galaxy and in the universe, is neither stationary (i.e. constant in time) nor homogeneous (i.e. uniform in space). This may be due to evolutionary aspects (both astrophysical and biological) or to uneven spatial distribution, both intrinsic (e.g. the existence of a galactic habitable zone) and arising from observational limitations (e.g. the interplay between detectors sensitivity and strength of the emitters). Such spatiotemporal aspects cannot be easily taken into account by the Drake equation in its standard form. We show that this may significantly impact on the estimates of the number of communicating civilizations of which we might find empirical evidence. We propose a simple and general framework to perform statistical simulations of the abundance of such civilization in an evolutionary scenario.

Key words. Astrobiology – Extraterrestrial life – SETI – Complex life – Life detection – Intelligence

1. Introduction

One of the main areas of investigation within SETI has always been the attempt to estimate the abundance of communicating civilizations in the universe. This is important not only in order to quantify the odds of success of SETI, but also to design and plan the best observational strategies. The usual approach to address this task is to start from the equation first proposed by Frank Drake in 1961(Drake 1961). As it is well known, the output of the Drake equation is a statistical estimate of the number of technological civilizations currently present in a certain volume of space (usually assumed to coincide with our Galaxy). This is based on a number of factors which quantify the influence of astrophysical, biological and sociological processes on the final outcome. Such factors are either unknown or known with some uncertainty. The utility of the Drake equation, historically, has been precisely to focus the discussion on the best ways to improve our knowledge on the relevant aspects of the problem, as well as on underlining its many facets.

There is no way to underestimate the importance of the Drake equation for the SETI community, and its role in steering most of the debate that took place over the past five decades. However, the equation also has some limitations, that suggest some caution in its uncritical application. One known shortcoming is its difficulty in dealing with the evolutionary aspects of SETI, i.e. with the possible time dependence of the abundance of civilizations (Ćirković 2004). Also difficult to treat is the possible spatial inhomogeneity of the distribution of civilization over the volume under exam. Many authors have discussed how the probability of the appearance of life may vary over the course of cosmic history (Loeb et al. 2016; Dayal et al. 2016) and how life might no be spread uniformly in the Galaxy (Gonzalez 2005; Lineweaver 2004) and it is therefore interesting to ask how these spatiotemporal dependencies should enter the estimate of the number of communicating civilizations in the universe. Furthermore, it is to be emphasized that we are not just interested in the number of civilizations existing at any time during the past, but we are looking for empirical evidences of their communications. Therefore considerations on the causal structure of the problem must also enter the calculations. Neglecting such effects might result in misestimates of the number of communicating civilizations and therefore of the chances of getting direct scientific evidence of the existence of intelligent life in the universe.

We have therefore proposed in a recent paper (Balbi 2018) a statistical framework that could easily incorporate the spatiotemporal aspects of SETI, allowing for their investigation. Here we summarize our main findings and suggest some possible directions for future studies.

2. Evolutionary effects and the Drake equation

Before illustrating our approach, it is useful to remind the properties of the Drake equation and to show why its standard form is not well-suited to treat spatiotemporal dependencies. The equation is usually written as:

$$N = R_\star f_p n_e f_l f_i f_c L \tag{1}$$

where N is the number of civilizations (in our galaxy or in a generic volume of space) capable of communicating over interstellar distances. The factors on the right hand side are the variables that have to be guessed or measured in order to perform the calculation: $R_\star$ is the star formation rate, f_p is the fraction of stars hosting planets, n_e is the number of habitable planets per star, f_l is the fraction of habitable planets where life actually appears, f_i is the fraction of inhabited planets where life evolves intelligence, f_c is the fraction of planets where intelligent life develops the necessary technology for interstellar communications. Finally, L is the average length of the communicating period of civilizations. Note that there is no explicit time dependence in the equation, although clearly there is no guarantee that the factors remained constant over time. As an obvious example, the rate of star formation certainly has changed over cosmic history. In fact, though not often stated, the Drake equation explicitly deals with *average* quantities. For example, the resulting value of N should be interpreted as the time-averaged number of communicating civilizations existing in the galaxy.

The fact that the Drake equation can at best deal with average quantities is apparent when one notices that the equation is just a specific example of a remarkable result of queueing theory, the so-called Little's law (Little 1961):

$$N = \lambda L \tag{2}$$

where N is the long-term average number of items in a stationary system (for example customers waiting to be served) is equal to the long-term average effective arrival rate λ of the items multiplied by the average time L that an item spends in the system. Clearly, in our case the system is the Galaxy (or more generally a volume of space around our location) and the average arrival rate is simply $\lambda = f_p n_e f_l f_i f_c$ (i.e. the apperance rate of technological civilizations), while L is the average communicating time. Little's result does not depend on the statistical distribution of the variables involved, but does assume that the underlying process is *stationary* (Little & Graves 2008). Thus, it cannot treat processes whose arrival rate is expected to vary in time, although there have been attempts to derive a time-varying versions of the law (Kim & Whitt 2013).

3. A framework to deal with the spatiotemporal aspects of SETI

We suggested elsewhere (Balbi 2018) that a simple framework to deal with a time-varying appearance rate of civilizations can be built upon two simple ingredients:

1. a causal constraint, i.e. the requirement that we can detect the interstellar communications of civilizations located at a given distance from us;

2. a statistical model of the spatial and temporal distribution of communicating civilizations.

Let us have a look at each in more detail.

3.1. Causal constraint

The causal constraint is simply defined by the fact that any message cannot propagate faster than light. If a civilization is located at distance D from us, its electromagnetic signals reach us after a travel time D/c. Now, imagine that the civilization started communicating at time t_c (where time is positive and growing towards the past, and the present epoch is taken as $t = 0$) and continued to do so uninterruptedly for a time τ. It is easy to show (see Figure 1) that we can receive its communications at present time if and only if:

$$t_c - \tau \leq D/c \leq t_c \tag{3}$$

This already has some interesting implications, because it introduces a correlation between the otherwise independent variables t_c (appearance time) and τ (longevity). Since the maximum distance, from our location, of other stars in our galaxy is of $\sim 10^5$ ly, the appearance time and longevity of any civilization that can communicate with us have to match to within 10^5 years, a time scale that is 4 to 5 orders of magnitude smaller that the age of the oldest stars in the Galaxy. This is a rather strict constraint, even in absence of any knowledge on the distribution of the time of appearance of civilizations. For example, it shows that any galactic civilization that ceased communicating (either intentionally or not) before $\sim 10^5$ years ago is undetectable by us. If the maximum distance covered by a SETI survey is smaller than the size of the Galaxy (a realistic depth reachable with present technology is of order $\sim 10^3$ ly), the required fine-tuning between t_c and τ is even stronger. This suggests that the statistical distribution of t_c will strongly influence the number of civilizations that can communicate with us at present time.

Evolutionary processes also enter in the analysis via the extent of the volume under investigation: more distant locations will probe more ancient epochs of cosmic history, which may correspond to varying abundances of civilizations. This is not accounted for in the Drake equation, which essentially produces a snapshot of the present situation (and implicitly assumes that it reflects the average situation at all cosmic epochs). The effect will be generally negligible for searches conducted within our galaxy, but might have important consequences if SETI is extended to extragalactic scales. In fact, exploring distances $D \sim 10^9$ Gy would result in much higher chances of picking up communications from ancient short-lived civilizations, whose signals have been traveling for a large fraction of cosmic history. Conversely, if we only search in our galactic neighbor the causal constraint would select civilizations with $t_c \sim \tau$. Whether this fine-tuning is a frequent or rare occurrence depends on the specific statistical distribution of these two variables. For example, one can assume that the appearance time t_c is uniformly distributed over the age of our galaxy, so that $t_c \sim 10^9$ years on average: in this case we should expect that any civilization of which we might find empirical evidence should either be almost exactly coeval with us or very long-lived. On the other hand, there might be a preferred time for the appearance of life in the galaxy, which can either be very close to the present epoch or very far back in the past: the two distributions of t_c would result in completely different estimates of the number of civilizations which we can detect.

3.2. Statistical model

To quantitatively explore the dependence on spatiotemporal assumptions, we can use the causal constraint of Eq. 3 as a filter that, given a population of N_{tot} communicating civilizations spread over the volume and history of the galaxy, selects only those that can be detected by us, N_D, thereby defining a "detectable fraction" $f_D = N_D/N_{\mathrm{tot}}$. Such filter can then be coupled to a model of the spatiotemporal distribution of communicating civilizations, in order to perform realistic simulations of the output of a SETI survey. For example, we might imagine that the distance D has either some

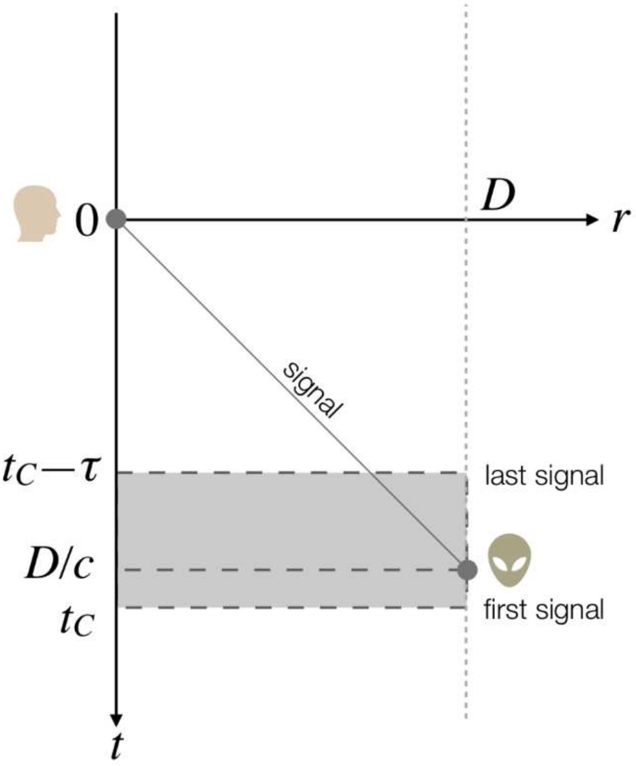

Fig. 1. The figure represents the causal requirement that the electromagnetic signals emitted by any communicating civilization has to satisfy in order to be received by us at present time. We are located at the origin of the spacetime coordinates. The communicating civilization is located at radial distance D from us and starts emitting signals at time t_c in the past (appearance time), for a total time interval τ (longevity). In order for the signal to be detectable by us today, the distance of the civilization, its appearance time and its longevity have to satisfy Eq. 3.

limit due to the depth of the survey, or a preferential value due to some intrinsic feature of the probability of appearance of life within the Galaxy, for example because of the existence of a Galactic habitable zone (Gonzalez 2005; Lineweaver 2004). Also, we can assume

Table 1. Detectable fraction f_D of communicating civilizations, for various distributions of t_c and τ. Each column corresponds to a choice of the average longevity, $\langle\tau\rangle$, assuming an exponential distribution. Each line corresponds to a different choice for the distribution of t_c. Distances D are uniformly distributed in a sphere of radius 10^3 ly around our location

	$\langle\tau\rangle = 10^3$ y	$\langle\tau\rangle = 10^6$ y	$\langle\tau\rangle = 10^9$ y
normal, $\langle t_c\rangle$=1 Gy, σ_{t_c}=2 Gy	$f_D \sim 10^{-7}$	$f_D = 2.5\ 10^{-4}$	$f_D = 0.26$
normal, $\langle t_c\rangle$=4 Gy, σ_{t_c}=2 Gy	$f_D = 0$	$f_D = 2.7\ 10^{-5}$	$f_D = 0.07$
normal, $\langle t_c\rangle$=4 Gy, σ_{t_c}=1 Gy	$f_D = 0$	$f_D = 10^{-7}$	$f_D = 0.03$
uniform (Drake)	$f_D = 0$	$f_D = 10^{-4}$	$f_D = 0.1$

a probability distribution for t_c and τ, reflecting some model of the temporal features of the appearance and disappearance of life and intelligence in the Galaxy.

As an illustration, in Balbi (2018) we simulated a population of $N_{\rm tot} = 10^7$ civilizations spread over Galactic history, and computed the fraction f_D that might be detected at present time. We assumed a uniform distribution of communicating civilizations in a spherical volume of radius 10^3 light years surrounding our location, and chose random distances D accordingly. Then, to each civilization we assigned a random t_c and τ drawn from some probability distribution. The longevity variable was always assumed to have an exponential probability distribution, with various average vales $\langle\tau\rangle$. The probability distribution of the appearance time t_c was chosen in order to reflect different hypotheses on the preferred epoch for the appearance of communicating civilizations. One case (normal distribution with average $\langle t_c\rangle$=1 Gy, σ_{t_c}=2 Gy) was chosen to match estimates of the most likely epoch of appearance of complex life in our galaxy(Lineweaver 2004). Other two cases assumed an early-appearance scenario where our species is a late-comer (normal distribution with average $\langle t_c\rangle$=4 Gy, σ_{t_c}=2 Gy and 1 Gy). Finally, one case assumed a uniform distribution for t_c over the age of our Galaxy ($T_G \sim 10^{10}$ y), meaning that there is no epoch more likely than others for the appearance of intelligent life. This is the case that can be directly compared to the Drake equation, in which case $\lambda = N_{\rm tot}/T_G$ and $f_D = \langle\tau\rangle/T_G$. Our results, summarized in Table 1, showed that, as expected, there is a significant dependence on the assumed distribution of t_c. In other words, for the same total number of civilizations that appeared over the course of cosmic history, the way they are spread in time (for example whether they appeared uniformly, or preferentially around a certain epoch) may result in a big difference in the number of civilizations that are in causal contact with us today. This, in turn, has a strong impact on the average communicating longevity $\langle\tau\rangle$ that is required in order to have at least another civilization within our observable volume, that can be as low as 10^3 or as high as 10^8 years, depending on the distribution of t_c.

4. Conclusions

Evolutionary effects are likely to play a role in estimating the distribution and abundance of communicating civilizations of which we might find empirical evidence. We showed that incorporating spatiotemporal effects in the Drake equation is not straightforward, and we proposed an alternative approach which has the benefit of generality and simplicity. What is required is essentially a model of the spatiotemporal distribution of communicating civilizations (their distances D, their appearance time t_c and their communicating longevity τ), which is then used to simulate their capability of being detected by their electromagnetic emission, through the fulfillment of a causal requirement. This model can be obtained, for example, through astrophysical or biological studies, either observational or theoretical, and can be used to make predictions of the odds of success of SETI. Arriving at such a model is a huge task, but the procedure outlined here can provide the proper framework for its development. Several other future developments of the work presented here are possible. They should include, for example, a realistic treatment of observational effects, such as the detector sensitivity, which impacts on the depth of survey, as well as a modeling of the strength and directional features of the emitters. Also, a realistic treatment of the spatial distribution of civilizations, based for example on a model of the galactic habitable zone, should enter the calculation.

References

Balbi, A. 2018, Astrobiology, 18, 54

Ćirković, M. M. 2004, Astrobiology, 4, 225

Dayal, P., Ward, M., & Cockell, C. S. 2016, `arXiv:1606.09224`

Drake, F. D. 1961, Physics Today, 14, 40

Gonzalez, G. 2005, Origins of Life and Evolution of the Biosphere, 35, 555

Kim, S.-H. & Whitt, W. 2013, Probability in the Engineering and Informational Sciences, 27, 471

Lineweaver, C. H. 2004, Science, 303, 59

Little, J. D. C. 1961, Operations Research, 9, 383

Little, J. D. C. & Graves, S. C. 2008, Operations Management, 115, 81

Loeb, A., Batista, R. A., & Sloan, D. 2016, J. Cosmol. Astropart. P., 08, 040